Two-body scattering revisited

a) **Matrix Element Practice**
Consider a theory with a Dirac fermion DM particle χ and spin $1/2$ nucleus N that we will treat as an elementary fermion of mass m_N. Suppose these interact through the effective operator

$$-\mathscr{L}_{eff} = \frac{1}{\Lambda^2}\,(\bar{\chi}\,i\gamma^5\chi)\,(\bar{N}N)\ , \tag{1}$$

where Λ has dimensions of mass.

a) Compute the summed and squared matrix element for $\chi(p_1) + N(p_2) \to \chi(p_3) + N(p_4)$.

b) Evaluate this in the lab frame with N initially at rest and the incident χ highly non-relativistic. Assume that $m_N \sim m_\chi$ and expand your result to leading order in v.

c) Compute the summed and squared matrix element for $\chi(p_1) + \bar{\chi}(p_2) \to N(p_3) + \bar{N}(p_4)$.

d) Evaluate this in the CM frame where both the incident χ particles are highly non-relativistic. Keep the N mass as well.

b) **Phase Space Practice**
Suppose our theory contains a pair of complex scalars with the interaction

$$-\mathscr{L} \supset \lambda\,|\phi|^2|\Phi|^2\ . \tag{2}$$

Take ϕ to be massless and Φ to have mass M.

a) Find the summed and squared matrix element for $\phi(p_1) + \Phi(p_2) \to \phi(p_3) + \Phi(p_4)$.

b) Suppose a ϕ with initial three-momentum $\vec{p} = p\,\hat{z}$ collides elastically with a Φ particle at rest. After the collision, the three-momentum of the outgoing ϕ will be $\vec{p}_3 = (p' s_\theta, 0, p' c_\theta)$. Apply energy and momentum conservation to find p' in terms of M, p, and θ.

c) Write an expression for the total cross section in this frame, but leave it as an expression with integrals over dp' and dc_θ and an energy-conserving delta function.

d) Use the result from c) to compute $d\sigma/dp'$.

e) Use the result from c) to compute $d\sigma/dc_\theta$.