Estimating Equilibrium

0 Background

In cosmology estimating the abundance of the particle species i involves solving the Boltzmann equation:

$$\frac{dn_i}{dt} + 3Hn_i = -\langle \sigma v \rangle (n_i^2 - n_{i\,\mathrm{eq}}^2). \tag{1}$$

However, before one embarks upon the task of solving this equation rigorously or numerically, it is useful to be able to make rough estimates using dimensional analysis. In this tutorial we shall use dimensional analysis to estimate the equilibrium conditions for different processes.

For this purpose it is useful to remember the following facts:

a) Know how to estimate $\langle \sigma v \rangle$.

b) Recall the number densities of particles in different limits

$$n_{i\,\mathrm{eq}} \sim \begin{cases} T^3 & \text{for } T \gg m_i \\ (m_i T)^{3/2} e^{-m_i/T} & \text{for } T \ll m_i \end{cases}$$

c) Assuming adiabatic expansion the total entropy, which is given by $S = sa^3$, where s is the entropy density and a is the scale factor, remains constant.

1 Massless Mediator

Consider the interactions between fermions f and ψ and the boson ϕ:

$$-\mathscr{L} \supset y_f \phi \bar{f} f + y_\psi \phi \bar{\psi} \psi \tag{2}$$

with $y_f \sim 1$ and $y_\psi \ll 1$, and all particles effectively massless.

a) What are the mass dimensions of y_f and y_ϕ?

b) Treating ϕ, ψ, and f as massless, estimate the cross section for $f\bar{f} \to \psi\bar{\psi}$ scattering at temperature T. (Just use dimensional analysis.) What is the corresponding rate for ψ production if the f are thermalized.?

c) Use this result to estimate the temperature range at which the density of ψ reaches equilibrium if one starts from a thermal plasma containing only f and $\bar{f}$ particles. Call the corresponding temperature T_{hi}.

2 Massive Mediator

Take the same interactions between massless ψ and f as above, but assume now that ϕ has a mass $m_\phi \ll T_{hi}$.

a) Convince yourself that at energies much less than m_ϕ, any interaction between ϕ and f mediated by ϕ can be reproduced by the effective interaction

$$- \mathscr{L}_{eff} \supset -\frac{y_f y_\psi}{m_\phi^2} \bar{\psi}\psi \, \bar{f} f \ . \tag{3}$$

b) Suppose we start with a cosmological plasma of f and ψ (and equal numbers of $\bar{f}$ and $\bar{\psi}$) at a temperature $m_\phi < T < T_{hi}$. As the plasma expands and cools, the temperature will eventually fall below m_ϕ. What is the effective interaction rate between ψ and f (using dimensional analysis)?

c) When does this interaction cease to equilibrate ψ and f? Call the corresponding temperature T_{lo}.

3 Freeze Out

Take the same interactions as before between ψ and f, and think of f as a SM fermion and ψ as the DM particle. However, suppose now that ψ is massive with $T_{hi} \gg m_\phi \gg m_\psi$.

a) Estimate the relic density of ψ if $m_\psi \ll T_{lo}$.

b) Do the same for $m_\psi \gg T_{lo}$.

c) In the non-relativistic freeze out case, when is kinetic equilibrium lost?

4 Neutrino Decoupling

Using the fact that neutrinos in the early universe annihilated into electron-positron pairs though Z bosons, find the temperature T at which neutrinos decoupled from the thermal equilibrium. Derive from this expression the neutrino temperature in today's universe compared to the cosmic microwave background.