Lecture Note #7: Dark Matter and Stars

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If dark matter (DM) interacts with visible matter, it can be captured or created in stars. This can potentially modify the behaviour of stars in observable ways. In these notes, we discuss some of the most promising possibilities.

1 How Stars Work

Stars are self-gravitating balls of gas supported by thermal pressure driven by nuclear reactions. Stars typically form from the mutual gravitational attraction of free gas, mostly hydrogen and helium. As this gas is compressed it heats up, and the temperature and pressure in the core can eventually become so large that nuclear fusion reactions start to occur. These reactions transform lighter nuclei into heavier ones and release energy into the stellar core. Eventually, the star can reach an approximately steady state where the thermal and gravitational pressures are balanced and the energy lost to radiation is matched by the energy created by nuclear fusion in the core.

The most common type of star, including our Sun, are *main sequence stars*. These are stars whose cores are supported mainly by the fusion of hydrogen into helium. The main sequence is typically the longest portion of the evolution of a star. Just how long it lasts depends mainly on the size of the star, which is traditionally measured in units of the solar mass, $M_\odot = 1.9891 \times 10^{30}\,\text{kg}$. Larger stars typically have higher temperatures and burn faster, corresponding to a higher *luminosity* and a shorter lifetime. This relationship is summarized in the Hertzprung-Russell (HR) diagram, shown in Fig. 1, which relates the observed surface temperature of stars to their net luminosity (in units of the solar luminosity).

A star will leave the main sequence when it runs out of hydrogen in its core. When this happens depends on the mass of the star, with larger stars burning out faster than smaller ones. For reference, the main sequence lifetime of a typical star with mass equal to $1\,M_\odot$ is about 10^{10} yrs, or about the age of the Universe.[1] Lighter stars burn more slowly, so they are usually expected to still be in the main sequence, while heavier stars may have transitioned to other stages of stellar evolution by now.

Evolution beyond the main sequence depends on the mass of the star. The helium produced by burning hydrogen is very tightly bound, and requires significantly higher temperatures to start fusion reactions of its own. For stars with $M \sim M_\odot$ this can lead to a bounce, where the core collapses on itself and rebounds when it is restabilized by the degeneracy pressure of electrons. The bounce pushes out material from the envelope of the star and can induce the remaining hydrogen outside the stellar core to ignite. The larger, cooler star that results is called a *red giant*. When the remaining hydrogen is used up, a second bounce occurs. For $M \lesssim 0.5\,M_\odot$, there is no reignition and the result is a core

[1] Don't worry. The Sun is thought to be about 5×10^9 yrs old, so we still have lots of time.

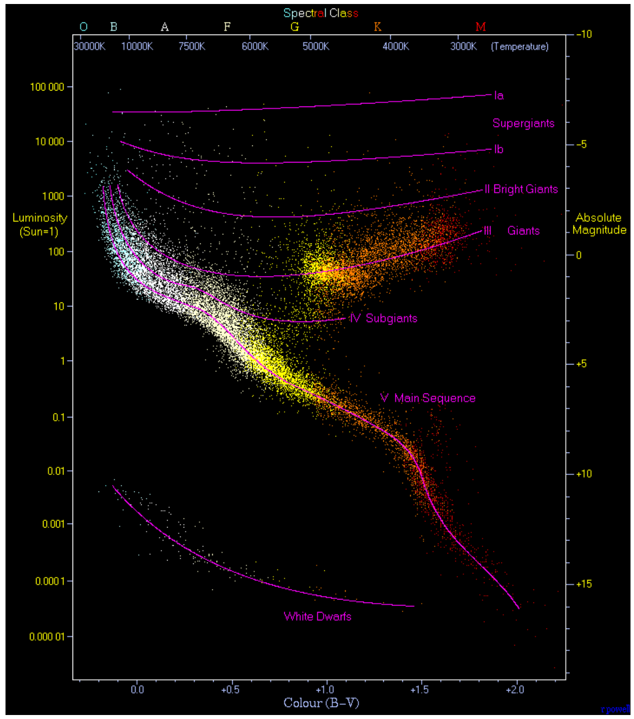

Figure 1: A Hertzprung-Russel diagram showing stellar lumisosities relative to their temperature (corresponding to their observed colour).

supported by electron degeneracy pressure surrounded by diffuse envelope of hot dust called a *planetary nebula*. When $M \gtrsim 0.5\,M_\odot$, this second bounce can ingnite helium fusion to produce an *asymptotic giant* star. This picture can repeat itself going to heavier elements, with helium burning replaced by the fusion of carbon, oxygen, or heavier elements, or it can terminate along the way with an inert core surrounded by a planetary nebula. The former is more likely for heavier stars, and the latter for lighter stars.

In very heavy stars, $M \gtrsim 10\,M_\odot$, helium fusion may begin before or very soon after the bounce, and continue on to heavier elements. When the reactions produce elements all the way up to iron in the core, no more energy can be gained from nuclear fusion. As can be seen in Fig. 2, ^{56}Fe has the highest binding energy per nucleon. This implies that for elements heavier than ^{56}Fe, it is energetically favourable to split into lighter nuclei by nuclear fission. Conversely, it is energetically favourable for lighter elements to fuse into heavier ones. After passing through the red giant and asymptotic giant phases all the way up to iron, a very heavy star will then collapse once and for all. There will again be a bounce, from the degeneracy pressure of electrons or neutrons. The case of a bounce stabilized by neutron degeneracy is especially violent, and causes a big explosian called a *supernova*. The conditions in a supernova shock wave are so extreme that elements heavier than iron can be created.

A *white dwarf* (WD) is the remnant of a stellar core supported by electron degeneracy pressure. They are formed by bounces that are unable to reignite nuclear fusion, and produce

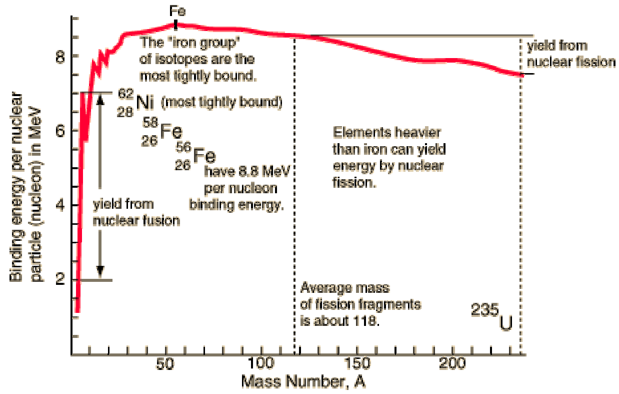

Figure 2: Plot of the binding energy per nucleon in nuclei as a function of the atomic mass number A. Energy can be gained by fusion from elements lighter than ^{56}Fe, and from fission for elements heavier that it. Figure from Ref.[1].

instead a planetary nebula and a WD core. This core is basically inert, and just cools as it ages. When a stellar core is heavier than the Chandrasekhar limit, $M \gtrsim 1.4\, M_{\odot}$, the electrons become relativistic below the Fermi momentum and their degeneracy pressure is no longer able to stabilize the core against collapse. Instead, the electrons combine with protons at high pressure to form non-relativistic neutrons, whose degneracy pressure can produce a stabilizing effect. If this pressure is enough, the result is a *neutron star* (NS). Like a WD, a neutron star is basically inert and just cools in time. For very large stellar cores, above about $M \gtrsim 3\, M_{\odot}$, not even the neutron pressure is enough and the core will collapse into a *black hole*.

2 Dark Matter Capture and Annihilation

Dark matter can be captured in stars by colliding elastically with the nuclei that make them up. To be captured, the DM velocity after scattering must be smaller than the escape velocity from the star (at the collision location). Once captured, a DM particle will typically undergo further scattering with stellar nuclei and thermalize to the temperature of the stellar interior while also being attracted towards the stellar core. The net result is spherical DM core, with density usually much smaller than the stellar core density, and a radius of

$$r_{th} \sim \left(\frac{T_c M_{\rm Pl}^2}{m_\chi \rho_c}\right)^{1/2} , \qquad (1)$$

where T_c is the temperature of the stellar core and ρ_c is its mass density. The DM density can grow large enough in the thermal core for DM particles to begin to annihilate. For WIMP-like DM, this annihilation can eventually balance the rate of capture allowing the total number N of DM particles in the star to reach a steady state.

The time evolution of the total number N of DM particles in a star is described by the simple equation

$$\frac{dN}{dt} = C - AN^2 \ , \tag{2}$$

where C is the total capture rate, and A is related to the DM annihilation rate in the stellar interior.

Capture rates of DM on a star depend on the local DM flux, the total effective scattering cross section, and the probability of the DM particle to exit the collision with a velocity below the local stellar escape velocity. An approximate expression for the capture rate is [2]

$$C = \frac{\rho_\chi \bar{v}}{m_\chi} \left(\frac{3v_{esc}^2}{2\bar{v}^2} \right) \sigma_{eff} \ , \tag{3}$$

where ρ_χ is the DM energy density at the location of the star, $\bar{v}$ is the typical local DM velocity, v_{esc} is the escape velocity from the stellar core, and σ_{eff} is an effective total cross section. The first factor in Eq. (3) is the local DM flux, the second factor accounts for the probability for a single collision to give an outgoing DM velocity below v_{esc}, and the last term is the effective cross section. This expression is a very crude approximation to the much more complicated full expression in Ref. [3].

The effective cross section of Eq. (4) has two possible forms, given by [2]

$$\sigma_{eff} = min \left\{ \sum_i \frac{M_*}{m_p} \frac{X_i}{A_i} \bar{\sigma}_{N_i}, \ \pi R_*^2 \right\} \ . \tag{4}$$

The first corresponds to a sum over all the scattering rates summed on all the nuclear constituents i of the star, with X_i the mass fraction of each species, A_i the atomic mass, and $\bar{\sigma}_{N_i}$ a typical χ-nucleus scattering cross section.[2] It applies in the *optically thin* limit, when a typical DM particle incident on the star is very unlikely to scatter to below v_{esc} at all. The second form in Eq. (4) corresponds to the *optically thick* limit, in which a DM particle incident upon the star is almost certain to be captured. On physical grounds, the total cross section cannot be too much larger than the cross-sectional area πR_*^2 of the star itself (where R_* is the stellar radius).

The annihilation coeffcient A in Eq. (2) is given by

$$A = \langle \sigma v \rangle_{ann} / (4\pi r_{th}^3/3) \ , \tag{5}$$

which is just the thermal annihilation cross section at temperature T_c (the temperature of the stellar core) divided by the volume of the thermal DM core.

Putting these pieces together, it easy to solve for the time evolution of the number of DM particles in the star, N:

$$N(t) = \sqrt{\frac{C}{A}} \tanh(t/\tau_*) \ , \tag{6}$$

[2] Note that since σ_i can increase quickly with A_i, up to A_i^4 in some cases, heavier elements that make up only a tiny fraction of the stars mass can sometimes dominate the capture rate.

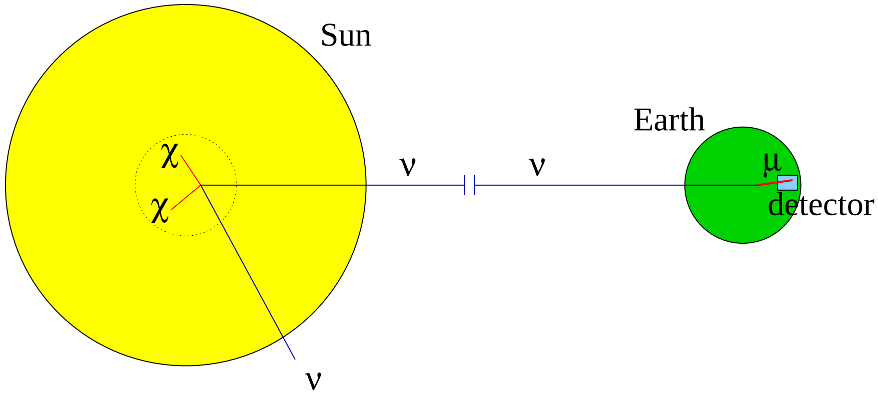

Figure 3: Illustration of an upward-going muon flux from DM annihilation in the Sun that produces energetic neutrinos. These can scatter inelastically off material in the Earth to create an upward-going flux of neutrinos that can be seen in underground detectors.

where $\tau_* = 1/\sqrt{CA}$ corresponds to an equilibration time. For $t \gg \tau_*$, the number of DM particles reaches a steady state $N = N_{eq} = \sqrt{C/A}$ where the rate of DM capture balances against the rate of DM annihilation.

Another possible effect that we have not yet included is *evaporation*, where DM particles captured thermally scatter to above v_{esc} and escape from the star [3]. This process would add a term of the form $(-EN)$ to the right side of Eq. (2). Evaporation is negligible for heavier DM or larger stars, but it can be important for lighter DM ($m_\chi \lesssim 4$ GeV in the Sun) with small annihilation cross sections. We will not worry about it here.

3 The Sun

Our very own Sun is one of the best places to look for DM. By mass, the Sun is mostly hydrogen and helium, but it also contains smaller amounts of heavier elements like carbon, oxygen, and iron that can be important for DM capture. WIMP DM is likely to reach a steady state by now, with $t_\odot > \tau_\odot$ for

$$\langle \sigma v \rangle_{ann} \gtrsim (3 \times 10^{-30} \mathrm{cm}^3/s) \left(\frac{\mathrm{GeV}}{m_\chi} \right)^{1/2} \left(\frac{10^{-40}\mathrm{cm}^2}{\sigma_p} \right) , \tag{7}$$

where σ_p is the effective scattering cross section on protons. Equilibration can be delayed, however, if the annihilation is dominantly p-wave.

The most promising signal of DM annihilation in the Sun is the emission of energetic neutrinos. Many DM annihilation modes create neutrinos, either directly or in the subsequent decays of the annihilation products such as bottom quarks or W bosons. For WIMP DM, these neutrinos will have energies in the multi-GeV range, which is much more energetic than other neutrino sources such as nuclear reactions in the Sun or the majority of cosmic ray neutrinos. This signal is searched for in neutrino detectors such as Super Kamiokande

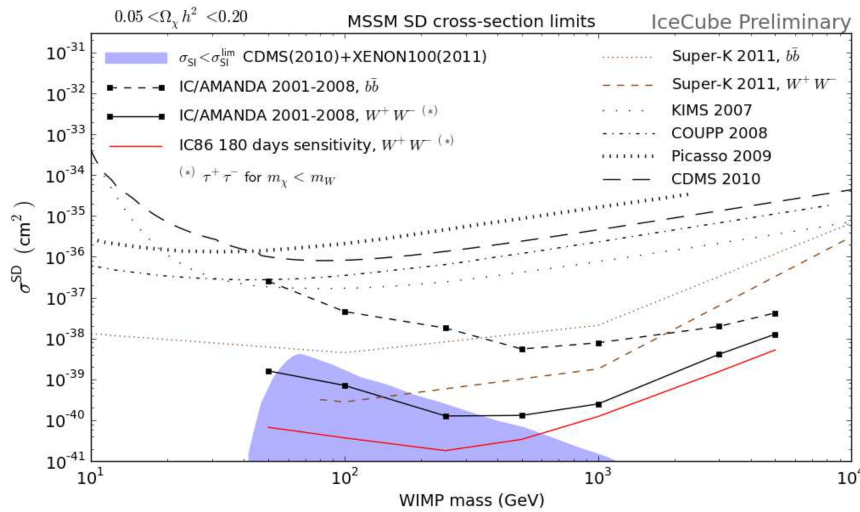

Figure 4: Limits on the effective spin-dependent proton cross section for DM scattering from direct detection searches as well as from indirect searches for neutrino fluxes due to DM annihilation in the Sun. The indirect search limits depend on the annihilation mode of the DM, and several examples are shown. Figure from Ref. [4].

and IceCube in the form of an upward-going muon flux – high-energy neutrinos from DM annihilation will scatter inelastically off material just outside the detectors to create upward-going, high-energy muons. We illustrate this in Fig. 3.

So far, no excess muon flux has been observed. This can be translated into a limit on the DM-nucleon scattering cross section under the assumption that the capture and annihilation rates have equilibrated. When they do, $C = AN^2$, and the net flux is proportional to σ_{eff} (with the optically-thin limit being applicable here). In many cases, the indirect limits on DM annihilation in the Sun are stronger than the direct detection limits, particularly for the case where the scattering is mainly spin-dependent. We show these limits in Fig. 4 for different assumptions about how the DM particles annihilate.

4 Other Main Sequence Stars

The energy deposited in stars by the annihilation of captured DM can have a significant effect on stellar evolution. This is particularly important for stars located near the galactic centre, where the DM density can be much larger than in our local region. The net effect is very complicated, and a detailed study can be found in Ref. [5].

5 Neutron Stars and White Dwarfs

Neutron stars are self-gravitating systems stabilized against collapse by the neutron degeneracy pressure of their cores. A typical neutron star has a mass close to that of the Sun, but compressed into a radius of about $R_* \simeq 10\,\mathrm{km}$, corresponding to a density of about $\rho_c \sim 10^{14}\,\mathrm{g/cm^3}$. As they age, they cool off, but this cooling can be interupted by the annihilation of dark matter captured within them. A similar story applies to white dwarfs, which are supported by the degeneracy pressure of electrons.

Capture of DM is very efficient in neutron stars, with a typical rate of [2, 6]

$$C \simeq 10^{24}\,\mathrm{s}^{-1}\left(\frac{\rho_\chi}{\mathrm{GeV/cm^3}}\right)\left(\frac{100\ \mathrm{GeV}}{m_\chi}\right) f\ , \tag{8}$$

with $f = min\{\sigma_n/10^{-45}\mathrm{cm}^{-2},\,1\}$. Once captured, the DM thermalizes quickly and sinks to a core of radius

$$r_{th} \simeq 20\,\mathrm{cm}\left(\frac{T_c}{10^5\mathrm{K}}\right)\left(\frac{100\ \mathrm{GeV}}{m_\chi}\right)^{1/2} . \tag{9}$$

The equilibration of capture and annihilation typically also occurs quickly, so the total heating rate from DM annihilation is $m_\chi A N^2 = m_\chi C$. The net effect of this heating is that the neutron star will not be able to cool as much as it would have in the absence of DM. Instead, the temperature of the star will reach a constant value when DM heating and cooling balance each other, equal to

$$T_{eq} \simeq 4000\,\mathrm{K}\left(\frac{\rho_\chi}{\mathrm{GeV/cm^3}}\right)^{1/4} . \tag{10}$$

This implies that if we were to observe a sufficiently cold neutron star, we could put an upper limit on the DM density in that region.

6 Non-Annihilating Dark Matter in Stars

Dark matter can also have important effects on stars when it is not able to annihilate. This can occur when the annihilation cross is very suppressed at low temperatures, or in the asymmetric DM scenario discussed in `notes-4` where the relic density is set by an initial asymmetry between the DM particle and its distinct antiparticle. In both cases, DM is captured in stars where it simply builds up as time goes on.

If the DM density becomes large enough, it can begin to self-gravitate and collapse within the stellar core. This occurs when the total mass within the thermal DM sphere grows larger than the total nuclear mass there [7, 8],

$$m_\chi N_{SG} = \frac{4\pi}{3} r_{th}^3 \rho_c\ , \tag{11}$$

where ρ_c is the nuclear density in the stellar core, r_{th} is given by Eq. (1), and N_{SG} is the number of DM particles that have collected and thermalized. Unless the gravitational collapse of the DM particles is stabilized in some way, a black hole will form that would engulf the star and potentially destroy it. Thus, the observation of long-lived stars in regions of high DM density can place limits on non-annihilating DM scenarios [7, 8].

The strongest bounds of this sort come from neutron stars; the capture DM very efficiently and they are seen in regions where the DM density is thought to be very large. In a typical neutron star, self-gravitation sets in for [7, 8]

$$N_{SG} \simeq 10^{41} \left(\frac{100\ \text{GeV}}{m_\chi}\right)^{5/2} \left(\frac{T_c}{10^5\,\text{K}}\right)^{3/2} . \tag{12}$$

When $N > N_{SG}$ the DM density begins to collapse upon itself. unless there is a pressure to stabilize it.

Assuming no DM self-interactions, the possible stabilization of the DM core depends on whether the DM particle is fermionic or bosonic. In the fermion case, there is a degeneracy pressure that will stabilize the DM core against collapse up to the Chandrasekhar limit. For a typical neutron star, the critical number of DM particles above which this limit is exceeded is [7, 8]

$$N_{\text{Ch}} \sim \left(\frac{M_{\text{Pl}}}{m_\chi}\right)^3 \simeq 10^{51} \left(\frac{100\ \text{GeV}}{m_\chi}\right)^3 . \tag{13}$$

This number is much larger than N_{SG}, and comparing to the capture rate of Eq. (8) we see that the limit from black hole formation of fermionic non-annihilating DM is quite weak. For bosonic DM, there is no longer a degeneracy pressure but there is a weaker uncertainty pressure related to the formation of a Bose-Einstein condensate (BEC). The corresponding critical number for a neutron star above which collapse occurs is [7, 8]

$$N_{\text{BEC}} \sim \left(\frac{M_{\text{Pl}}}{m_\chi}\right)^2 \simeq 10^{34} \left(\frac{100\ \text{GeV}}{m_\chi}\right)^2 . \tag{14}$$

This number is smaller than N_{self}. Thus, when $N > N_{self}$, the core of bosonic non-annihilating DM will collapse on itself to form a black hole that can engulf and destroy the star. The collapse can occur even earlier from the formation of a BEC alone, but the resulting black hole may evaporate before destroying the star. Very old neutron stars have been observed in regions of significant DM density, and this puts strong limits on the nucleon scattering cross sections of bosonic non-annihilating DM [7, 8], as can be seen in Fig. 5.

It should be mentioned, however, that these limits can be avoided if the DM particle has even a very weak repulsive self interaction [9]. In this case, the pressure due to mutual repulsion can stabilize against collapse. Two more ways out are to have an alternative annihilation channel (such as annihilation with baryons) or from evaporation in the case of very light DM.

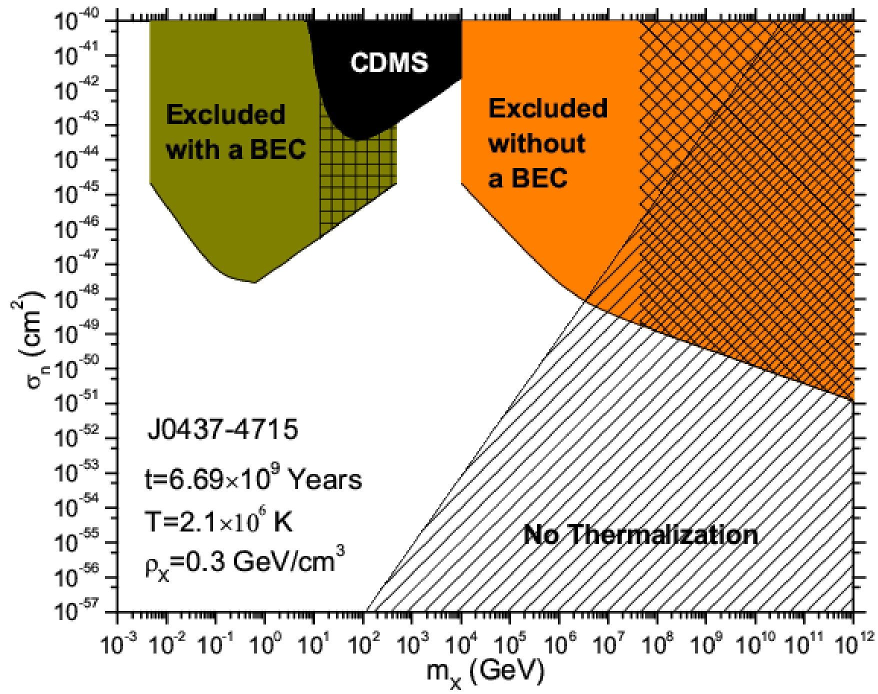

Figure 5: Limits on non-annihilating DM from black hole formation in the neutron star J0437-4715. Figure from Ref. [7].

7 Axions and Stars

Axions are another non-WIMP possibility for DM, and they can also affect stars in important ways. We will discuss the corresponding limits in `notes-9` when we get to describing axions in detail.

References

[1] See: http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin.html.

[2] G. Bertone, M. Fairbairn and , "Compact Stars as Dark Matter Probes," Phys. Rev. D **77**, 043515 (2008) [arXiv:0709.1485 [astro-ph]].

[3] A. Gould, "Wimp Distribution In And Evaporation From The Sun," Astrophys. J. **321**, 560 (1987).

[4] R. Abbasi *et al.* [IceCube Collaboration], "Multi-year search for dark matter annihilations in the Sun with the AMANDA-II and IceCube detectors," Phys. Rev. D **85**, 042002 (2012) [arXiv:1112.1840 [astro-ph.HE]].

[5] P. Scott, M. Fairbairn, J. Edsjo and , “Dark stars at the Galactic centre - the main sequence,” Mon. Not. Roy. Astron. Soc. **394**, 82 (2009) [arXiv:0809.1871 [astro-ph]].

[6] C. Kouvaris, “WIMP Annihilation and Cooling of Neutron Stars,” Phys. Rev. D **77**, 023006 (2008) [arXiv:0708.2362 [astro-ph]].

[7] S. D. McDermott, H. -B. Yu, K. M. Zurek and , “Constraints on Scalar Asymmetric Dark Matter from Black Hole Formation in Neutron Stars,” Phys. Rev. D **85**, 023519 (2012) [arXiv:1103.5472 [hep-ph]].

[8] C. Kouvaris, P. Tinyakov and , “Excluding Light Asymmetric Bosonic Dark Matter,” Phys. Rev. Lett. **107**, 091301 (2011) [arXiv:1104.0382 [astro-ph.CO]].

[9] C. Kouvaris, P. Tinyakov and , “On (Not)-Constraining Heavy Asymmetric Bosonic Dark Matter,” arXiv:1212.4075 [astro-ph.HE].