Lecture Note #3: Thermal Freeze Out and WIMPs

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March 25, 2013

One of the leading candidates for thermal DM is a weakly-interacting massive particle (WIMP). This is a new particle that is massive, stable, and neutral, and that interacts with the SM exclusively through the weak force (and possibly the Higgs). Such particles arise in many theories that extend the Standard Model (SM), and they can provide the correct DM density through thermal freeze out in a very natural way. For these reasons, they are by far the most popular class of DM candidates.

1 The WIMP Miracle

It is very instructive to apply our approximate expressions for the freeze out temperature and the relic density (Eqs. (26,29) in `notes-2`) to the case of a WIMP particle χ with a mass close to the electroweak scale. The first thing we need is the annihilation cross section. Without actually doing a calculation, we can estimate the parametric dependence of the cross section by counting couplings and applying dimensional analysis. The cross section has dimensions of $(mass)^{-2}$ and the largest dimensionful quantity around is the mass of the WIMP m_χ. The cross section should also contain at least four factors of the weak coupling constant $g \simeq 0.65$. Together, this gives

$$\langle\sigma v\rangle \sim \frac{g^4}{4\pi}\frac{1}{m_\chi^2} \simeq (1.7\times 10^{-23}\mathrm{cm}^3/s)\left(\frac{100\ \mathrm{GeV}}{m_\chi}\right)^2 \ . \tag{1}$$

This is very crude, but it will do for our purposes.

Plugging our estimate into Eq. (26) in `notes-2`, we get

$$x_f \simeq 27.9, \qquad \Omega_\chi h^2 \simeq 0.0002\ . \tag{2}$$

This is within a few orders of magnitude from what is needed to explain the observed DM density. Given all the factors that go into the relic density, from particle physics stuff like couplings and masses to cosmological quantities like the Hubble rate today, it is amazing that a generic WIMP is so close to the correct answer. This surprising result is called the *WIMP miracle* [1, 2, 3]. The motivation for WIMPs is strengthened even more by the fact that we have many other particle physics reasons (unrelated to cosmology) to expect new physics near the electroweak scale.

In passing, let us note that for $x_f \simeq 25-30$, the value one obtains for a broad range of DM masses and annihilation cross sections, the relic density is approximately

$$\Omega_\chi h^2 \simeq 0.1\,\frac{(3\times 10^{-26}\mathrm{cm}^3/s)}{\langle\sigma v\rangle}\ . \tag{3}$$

This is a useful benchmark against which to compare quick estimates of the annihilation cross section.

2 Popular WIMP Candidates

For many reasons we expect that there exist new particles and forces (beyond the SM) with masses near or the electroweak scale. The strongest reason to expect such new physics is the *electroweak hierarchy problem*, which is that the scale of electroweak symmetry breaking apears to be destabilized by quantum corrections. Proposals to solve this problem include supersymmetry (SUSY), extra dimensions, and new strong forces. The WIMP miracle described above gives a further piece of motivation for new physics at the electroweak scale. Indeed, many extensions of the SM contain (or can accommodate) a WIMP DM candidate.

2.1 WIMPs from Supersymmetry

The most popular extension of the SM that addresses the hierarchy problem is supersymmetry [4]. Exact supersymmetry predicts that every SM particle should have a *superpartner* with the same mass and quantum numbers, but with a spin differing by half a unit. For example,

$$\begin{array}{ccc} \text{fermion} & f \leftrightarrow \widetilde{f} & \text{sfermion} \\ (s=1/2) & & (s=0) \\ \\ \text{gauge boson} & A_\mu \leftrightarrow \widetilde{A} & \text{gaugino} \\ (s=1') & & (s=1/2) \\ \\ \text{Higgs} & H \leftrightarrow \widetilde{H} & \text{Higgsino} \\ (s=0) & & (s=1/2) \end{array} \tag{4}$$

The minimal supersymmetric extension of the SM (MSSM) has a superpartner for every SM particle, and basically nothing else. The lone exception is the Higgs sector, where two scalar $SU(2)_L$ Higgs doublets $H_{u,d}$ are required along with their Higgsino superpartners $\widetilde{H}_{u,d}$.

Supersymmetry stabilizes the electroweak scale by imposing a cancellation of quantum corrections to the Higgs fields (which induce electroweak symmetry breaking) between SM particles and their superpartners. The dangerous corrections cancel exactly if supersymmetry is an exact symmetry of Nature. However, this would also imply the existence of scalar electrons (selectrons) with the same mass as the electron, a possibility that is very firmly ruled out. On the other hand, if supersymmetry is broken the superpartners can be heavier than their SM counterparts. It turns out that broken supersymmetry can still protect the electroweak scale as long as all the operators that break SUSY have couplings of positive mass dimension that are not too large. This type of breaking is called *soft* because its effects become negligible at energies much larger than the scale of the SUSY-breaking couplings. By not too large, the quantitative requirement is $m_{soft} \lesssim 1000$ GeV, which implies that the superpartners must have masses close to this value. The LHC is currently probing this regime.

Even with soft breaking, the addition of superpartners to the SM can lead to all sorts of bad things happening, such as rapid proton decay, unless we also impose a further symmetry called R-parity.[1] This is a $\mathbb{Z}_2$ symmetry under which all the SM particles are even and all the superpartners are odd. As a result, superpartners must be created or destroyed in pairs, and the lightest superpartner (LSP) is stable.

The LSP can be a viable DM candidate if it is uncharged and uncoloured. In the MSSM, the two possibilities are the lightest the lightest sneutrino and the lightest neutralino. It turns out that the MSSM sneutrino as DM is ruled out by direct searches for DM (although it can work in extensions of the MSSM with additional gauge singlet fields). Thus, we will focus here on the case of a neutralino LSP.

There are four neutralino states in the MSSM, which we will denote by χ_i^0, $i = 1, 2, 3, 4$, with the labels such that the masses are increasing, $m_{\chi_1} \leq m_{\chi_2} \leq m_{\chi_3} \leq m_{\chi_4}$. These four neutralino states are linear combinations of the Bino ($\widetilde{B}^0$ = superpartner of the $U(1)_Y$ gauge boson), Wino ($\widetilde{W}^3$ = superpartner of the neutral component of the $SU(2)_L$ gauge boson), and the Higgsinos ($\widetilde{H}_u^0$, $\widetilde{H}_d^0$ = superpartners of the neutral components of the two Higgs scalar doublets). Thus, we have

$$\chi_i^0 = N_{i1}\widetilde{B}^0 + N_{i2}\widetilde{W}^0 + N_{i3}\widetilde{H}_d + N_{i4}\widetilde{H}_u \ , \tag{5}$$

where N_{ij} is a unitary mixing matrix. The mixing arises mostly from electroweak symmetry breaking. Note that the four neutralinos are all Majorana fermions, meaning that they are their own antiparticles.

In general, a mostly-Bino LSP annihilates inefficiently and produces too much DM through thermal freeze out, while a mostly Wino or Higgsino LSP produces too little. This is partly the result of the hypercharge gauge coupling being smaller than the $SU(2)_L$ coupling, and partly due to the absence of other nearly degenerate states that help the self-annihilation to be more efficient. An acceptable thermal relic density can be obtained, however, when the LSP is a roughly equal mixture of the Bino and the other states [5]. There are also some special cases where the annihilation of a Bino-like neutralino is enhanced and the relic density comes out right. We will discusses some of the ways this can happen below.

2.2 WIMPs from an Extra Dimension

A second way to address (or at least recast) the hierarchy problem is to postulate one or more extra dimensions of spacetime with a characteristic size of $R \sim \text{TeV}^{-1}$. Some of these theories come with a $\mathbb{Z}_2$ reflection symmetry that can give rise to a viable WIMP DM candidate.

The most popular example of this type of scenario is Universal Extra Dimensions (UED) [6]. In the simplest realization of UED, there is a flat fifth dimension of length $R \sim \text{TeV}^{-1}$ bounded on either end by four-dimensional surfaces called *branes*. This additional dimension is assumed to be symmetric under reflection about its midpoint, and all the fields of the SM are able to propagate within it. See Fig. 1 for an illustration.

[1] There are other possibilities as well, but R-parity is the simplest and most popular.

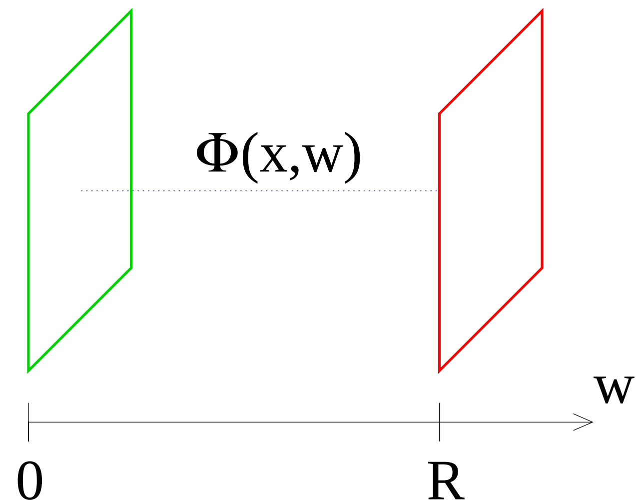

Figure 1: Cartoon of UED. The field Φ represents a species from the SM allowed to propagate in the fifth dimension.

Just like in four dimensions, the fundamental SM degrees of freedom (*i.e.* elementary particles) are derived from quantum fields. However, these fields are now defined everywhere in all five dimensions subject to boundary conditions at the brane boundaries. To illustrate this, consider a real scalar field $\Phi(x^\mu, w)$ subject to the boundary condition

$$\partial_w \Phi(x^\mu, w)|_{w=0,R} = 0 \ , \tag{6}$$

where $w \in [0, R]$ is the coordinate of the fifth dimension. Ignoring interactions, the Lagrangian of Φ is

$$S = \int d^4x \int dw \ \frac{1}{2} \left[(\partial_\mu \Phi)^2 - (\partial_w \Phi)^2 - M^2 \Phi^2 \right] \ . \tag{7}$$

We can recast this Lagrangian in a more familiar form by expanding the field $\Phi(x, w)$ according to

$$\Phi(x, w) = \frac{1}{\sqrt{R}} \sum_{n=0}^{\infty} \phi_n(x) \ \cos\left(\frac{n\pi}{R} w \right) \ , \tag{8}$$

for some coefficient fields $\phi_n(x)$. Note that any function of w on $[0, R]$ subject to the boundary conditions of Eq. (6) can be expanded in this way. Inserting Eq. (8) back into Eq. (7) and integrating over w gives

$$S = \int d^4x \sum_{n=0}^{\infty} \frac{1}{2} \left[(\partial_\mu \phi_n)^2 - \left(M^2 + \frac{n^2\pi^2}{R^2} \right) \phi_n^2 \right] \ . \tag{9}$$

In this form, we see that the five-dimensional field can be recast into an infinite tower of independent four-dimensional *Kaluza-Klein (KK) modes*, each with mass $m_n^2 = M^2 + (n\pi/R)^2$.

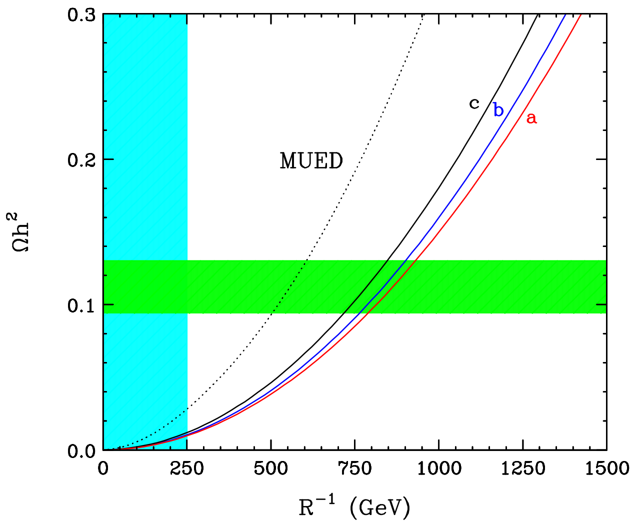

Figure 2: Relic density of the LKP in minimal UED, including the effects of coannihilations (dotted line). The other lines correspond to less complete calculations. Figure from Ref. [8]

This picture applies to all SM fields in UED. We identify the $n = 0$ *zero mode* with the regular SM particle, while the higher modes make up the KK excitations of that particle. The KK masses at level n are

$$m_n = \begin{cases} \sqrt{m_{SM}^2 + (n\pi/R)^2} & ; \quad \text{bosons} \\ \\ m_{SM} + (n\pi/R) & ; \quad \text{fermions} \end{cases} . \tag{10}$$

The KK partners of the SM are heavier due to the $(n\pi/R)$ contributions to their masses.

Interactions among SM fields and their KK partners are obtained by writing the usual SM couplings in terms of the five-dimensional fields. When the extra dimension has a reflection symmetry about its centre (as we assume in UED), these interactions respect a new $\mathbb{Z}_2$ symmetry called *KK parity*. This symmetry implies that in any allowed interaction the sum of the KK numbers of the fields making up the corresponding operator must be even. As a result, any interaction with only fields from the SM ($n = 0$) and the first KK level ($n = 1$) must involve an odd number of $n = 1$ states. The physical implication of this is that odd KK modes must be created in pairs, and that the lightest $n = 1$ KK particle (LKP) is stable. This makes the LKP a potential candidate for DM [7].

In the minimal realization of UED, the LKP is a mixture of the vector-boson KK partners of the hypercharge B_μ and $SU(2)_L$ W_μ^3 fields. This state is electrically neutral, and has a mass close to π/R. Calculations of the annihilation cross section of this state show that it gives the observed relic density through thermal freeze out provided its mass is close to $m_{LKP} = 500$ GeV [7, 8]. Other masses and types are LKP can arise in extensions of the minimal model. The dependence of the LKP relic density on its mass is shown in Fig. 2.

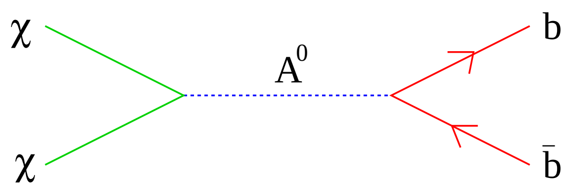

Figure 3: Feynman diagram for $\chi_1^0\chi_1^0 \to A^0 \to b\bar{b}$.

2.3 Other Possibilities

There are many other theories that predict WIMP (or WIMP-like) DM. In general, all that is needed is a symmetry to stabilize the DM candidate and a way for that particle to annihilate in the early Universe. The DM candidate may even be unstable, provided its lifetime is very long compared to the age of the Universe.

3 Special Cases for DM Annihilation

As mentioned above, a mostly-Bino LSP in supersymmetry tends to produce too much DM by thermal freeze-out. However, such a state can produce the correct amount of DM in certain special cases. We will describe two of the most popular examples here: resonant enhancement, and coannihilation [9]. While we focus on supersymmetry in this section, these mechanisms apply just as well to other DM candidates.

3.1 Resonant Enhancement

Consider the annihilation channel $\chi_1^0\chi_1^0 \to A^0 \to b\bar{b}$ by way of an intermediate s-channel pseudoscalar A^0 (that arises in the supersymmetric SM). We show the corresponding Feynman diagram in Fig. 3. The squared amplitude for the annihilation cross evidently contains an A^0 propagator, and it goes like

$$\mathcal{M} \propto \left|\frac{1}{s - m_A^2 + i m_A \Gamma_A}\right|^2 \tag{11}$$

where $s = (p_{\chi_1} + p_{\chi_2})^2 \simeq 4m_\chi^2$ and $\Gamma_A = \tau_A^{-1}$ is the decay width of the A^0 particle. Since $\Gamma_A \ll m_A$ in most cases, the annihilation cross section will be strongly enhanced for $m_\chi \simeq m_A/2$ corresponding to the intermediate A^0 being nearly on-shell. This can give a very large enhancement relative to the crude estimate of Eq. (1). When the resonance is very narrow, it is also important to integrate the Boltzmann equation numerically rather than to use the approximate method presented in `notes-2` [9].

3.2 Coannihilation

A second important special case is coannihilation. In writing the Boltzmann equation for the freeze out of χ in `notes-2`, we implicitly assumed that the only relevant particles were χ and those of the SM. In SUSY (and other scenarios), there may be other new particles that can also play an important role in determining the relic density. For example, χ_1^0 particles can be created through the decay $\widetilde{q} \to q\chi_1^0$ (where $\widetilde{q}$ is a squark), and they can annihilate via $\widetilde{q}\chi_1^0 \to q\,g$. Coannihilation refers to all these additional reactions that we have not yet considered.

In many situations the coannihilation reactions are not important and can be neglected. For example, suppose the squark $\widetilde{q}$ is much heavier than the χ_1^0 LSP. In this case the squarks will freeze out on their own and decay down to χ_1^0 well before χ_1^0 freezes out. Since χ_1^0 remains close to thermodynamic equilibrium before its own freeze out, the extra χ_1^0 particles produced by these decays will annihilate away as this state tracks its equilibrium value.

Coannihilation can be very important when the LSP is close in mass to one or more of the other superpartners [9]. (A good rule of thumb is that it can be important when the masses are within 25% or so). In this case it is no longer a good approximation to treat the freeze-out of χ_1^0 alone. Instead, the correct way to handle the other superpartners is to compute the total relic density of all the superpartners, $n = \sum_i n_i$, by combining the Boltzmann equations for all of them. Since each relic superpartner will either annihilate to the SM or eventually decay to the LSP, $n \to n_{SLP}$ at late times. The corresponding Boltzmann equation for n is [10]

$$\frac{dn}{dt} + 3Hn = \sum_{ij} \langle\sigma_{ij} v_{ij}\rangle \left(n_i n_j - n_{i_{eq}} n_{j_{eq}}\right) \ , \tag{12}$$

where the sum runs over all superpartners and σ_{ij} runs over all reactions of the form $(i+j \to SM)$. The thermal average is a simple generalization of what we had before:

$$\langle\sigma_{ij} v_{ij}\rangle = \frac{1}{n_{i_{eq}} n_{j_{eq}}} \int \frac{d^3p_i}{(2\pi)^2}\frac{d^3p_j}{(2\pi)^2}\, g_i g_j f_{i_{eq}} f_{j_{eq}} \sigma_{ij} v_{ij} \ , \tag{13}$$

and $v_{ij} = \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2}$.

The LSP relic density can be increased or decreased by coannihilation. As a specific example, suppose the annihilation cross section of χ_1^0 with itself ($\langle\sigma_{11}v_{11}\rangle$) is very small compared to the cross section for $\chi_1^0 + \tilde{q} \to SM$ ($\langle\sigma_{12}v_{12}\rangle$). In this case, as the χ_1^0 are in the process of freezing out, the scattering process $\chi_1^0 + q \to g + \tilde{q}$ can transfer some of the χ_1^0 density to a squark density. The squark produced in this way can now annihilate efficiently with the remaining neutralinos, and the net effect is to deplete the neutralino density more efficiently than self-annihilation alone. Note that for this mechanism to have a significant effect, the transfer reaction ($\chi_1^0 + q \to g + \tilde{q}$) must be efficient near the time of freeze out, and this requires that the mass of $\tilde{q}$ not be too much bigger than χ_1^0 so that thermal collisions in the plasma have enough energy to excite it.

Although we have illustrated coannihilation in the context of SUSY, it can also play an important role in other theories containing DM. A specific example is minimal UED, where many of the KK partners are very close in mass to the LKP. The leads to a large reduction of the final LKP density from coannihilation [8].

4 Variations on WIMPs

There are some simple variations on the WIMP picture for the formation of the DM density, and we present a couple of them here.

4.1 The WIMPless Miracle

The cross section in Eq. (1) goes like $(g_\chi^2/m_\chi)^2$. This gives about the right relic density for a WIMP, with $m_\chi \sim m_W \sim 100$ GeV and $g_\chi \sim g_{weak} \simeq 0.65$. However, we could also get about the right DM density from a particle that is much lighter but with much smaller couplings to the SM, such that the ratio $(g_\chi^2/m_\chi) \sim (1/100\text{ GeV})$ is about the same as for a WIMP.

This generalization is sometimes called the WIMPless miracle [11]. It can occur, for example, if DM couples to a new force that mediates its annihilation. For couplings weaker than weak ($g_\chi \ll g_{weak}$), DM particles with masses much lighter than the weak scale can give the correct relic density through thermal freeze out provided $(g_\chi^2/m_\chi) \sim (1/100\text{ GeV})$ still holds true. Interestingly, this relationship between the dimensionless coupling and the particle mass occurs automatically in supersymmetric theories where the breaking of supersymmetry is communicated to the DM particle through the new gauge interaction. [2]

4.2 SuperWIMPs

A second variation on the basic WIMP scenario are superWIMPs [12]. Suppose there exists a metastable WIMP ψ that undergoes thermal freeze out in the usual way, but later decays to a lighter stable particle χ and a SM particle, $\psi \to \chi + f$. If the χ particles have already frozen out when this occurs, these decays will provide an additional contribution to their relic density.

The net density of χ particles in this case will be

$$\Omega_\chi h^2 = \Omega_\chi^{(th)} h^2 + \left(\frac{m_\chi}{m_\psi}\right) \Omega_\psi h^2 \ , \tag{14}$$

where $\Omega_\chi^{(th)} h^2$ is the χ density produced by thermal processes (like its own freeze out), and $\Omega_\psi h^2$ is the relice density of ψ that would occur if this state did not decay.

[2] Mechanisms of communicating supersymmetry breaking will be discussed in the next set of notes.

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