Estimating Equilibrium

0 Background

In cosmology estimating the abundance of the particle species i involves solving the Boltzmann equation:

$$\frac{dn_i}{dt} + 3Hn_i = -\langle \sigma v \rangle (n_i^2 - n_{i\,\mathrm{eq}}^2). \tag{1}$$

However, before one embarks upon the task of solving this equation rigourously or numerically, it is useful to be able to make rough estimates using dimensional analysis. In this tutorial we shall use dimensional analysis to estimate the equilibrium conditions for different processes.

For this purpose it is useful to remember the following facts:

a) Know how to estimate $\langle \sigma v \rangle$.

b) Recall the number densities of particles in different limits

$$n_{i\,\mathrm{eq}} \sim \begin{cases} T^3 & \text{for } T \gg m_i \\ (m_i T)^{3/2} e^{-m_i/T} & \text{for } T \ll m_i \end{cases}$$

c) Assuming adiabatic expansion the total entropy, which is given by $S = sa^3$, where s is the entropy density and a is the scale factor, remains constant.

1 Dark Matter Equilibration

Let us consider a massless scalar field ϕ that interacts with some fermions f and itself via the Lagrangian

$$-\mathcal{L}_1 \supset y_f \bar{f} f \phi + g\phi^3$$

where g is a coupling constant of the order of unity.

- Starting from a plasma of ϕ particles at very high temperature estimate the rate at which such a plasma would create fermions. In other words compute $\langle \sigma v \rangle n_\phi$.
- At what temperature would the particle f reach equilibrium? [Hint: compare the rate you computed above to the Hubble rate.]

2 Dark Matter Decoupling

At temperature $T < M$ where M is the weak scale the effective interactions between dark matter particles χ and some standard model fermion f is given by

$$-\mathcal{L}_2 \supset \frac{1}{M^2}\bar{\chi}\chi\bar{f}f.$$

Derive the temperatures between which the dark matter particles remain in equilibrium.

3 Falling out of kinetic equilibrium

Under the same assumption as above derive the condition when dark matter particles fell out of kinetic equilibrium with standard model fermions.

4 The Neutrino Temperature

Using the fact that neutrinos in the early universe annhililated into electron-positron pairs though Z bosons, find the temperature T at which neutrinos decoupled from the thermal equilibrium. Derive from this expression the neutrino temperature in today's universe compared to the cosmic microwave background.