Direct Detection of Dark Matter

In dark matter detection we can write down the following possible effective interaction Lagrangians between the nuclei and the dark matter particles:

$$\mathcal{L}_1 \supset \frac{1}{M^2}\bar{\chi}\chi\overline{N}N$$
$$\mathcal{L}_2 \supset \frac{1}{M^2}\bar{\chi}\gamma^5\chi\,\overline{N}\gamma^5 N$$
$$\mathcal{L}_3 \supset \frac{1}{M^2}\bar{\chi}\gamma^5\chi\,\overline{N}N.$$

It was mentioned in the class the cross-sections calculated with the first two of these Lagrangians have different leading order behaviour in v than the last one. In this tutorial we shall explicitly see this.

Consider the situation when a dark matter particle χ with speed v smashes into a nucleus that is at rest in the lab frame.

- Show that the momentum imparted to the nucleus in such a collision is

  $$q = 2M_N\, v\cos\theta$$

  where $M_N = \frac{m_\chi m_N}{m_\chi + m_N}$ is the reduced mass and θ is the angle between the directions of the impinging dark matter particle and the recoiling nucleus,

- Write down cross-sections for this scattering process for the three Lagrangians listed above.

  [Recall that the differential cross-section of two particle scattering process is given by

  $$d\sigma = d\Phi\frac{|\mathcal{M}_{2\to n}|^2}{4\sqrt{(k_a\cdot k_b)^2 - m_a^2 m_b^2}}$$

  where k_a and k_b are the four-momenta of the incoming particles.]

- Express these cross section in powers of v^2 as would be relevant for non-relativistic dark matter particles.