# Lecture Note #3: More on Dark Matter Creation

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# 1 WIMPs and Thermal Freeze Out

One of the leading candidates for DM is a weakly-interacting massive particle (WIMP); that is a new particle that is stable and neutral, and that interacts with the SM exclusively through the weak force (and possibly the Higgs). We will discuss them in this section.

## 1.1 The WIMP Miracle

It is very instructive to apply our approximate expressions for the freeze out temperature and the relic density (Eqs. (26,29) in Note #2) to the case of a WIMP particle with a mass close to the electroweak scale. The first thing we need is the annihilation cross section. Without going into any details, we can estimate the parametric dependence of the cross section by counting couplings and applying dimensional analysis. The cross section has dimensions of $(mass)^{-2}$ and the largest dimensionful quantities around are $m_\chi \sim m_W$. The cross section should also contain at least four factors of the weak coupling constant $g \simeq 0.65$. Together, this gives

$$\langle \sigma v \rangle \sim \frac{g^4}{4\pi}\frac{1}{m_\chi^2} \simeq (1.7 \times 10^{-23} cm^3/s) \left(\frac{100\text{ GeV}}{m_\chi}\right)^2 . \qquad (1)$$

This is very crude, but it will do for our purposes.

Putting this in to Eq. (26) in Note #2, we get

$$x_f \simeq 27.9, \qquad \Omega_\chi h^2 \simeq 0.0002 . \qquad (2)$$

This is within a few orders of magnitude of what is needed to explain the observed DM density. Given all the factors that go into the relic density, from particle physics stuff like couplings and masses to cosmological quantities like the Hubble rate today, it is amazing that a generic WIMP is this close to the correct answer. This is sometimes called the WIMP miracle. The motivation for WIMPs is strengthened even more by the fact that we have many other particle physics reasons (unrelated to cosmology) to expect new physics near the electroweak scale.

In passing, let us note that for $x_f \simeq 25-30$, which is the value one obtains for a broad range of DM masses and annihilation cross sections, the relic density is approximately

$$\Omega_\chi h^2 \simeq 0.1 \, \frac{(3 \times 10^{-26} cm^3/s)}{\langle \sigma v \rangle} . \qquad (3)$$

This is a useful benchmark against which to compare quick estimates of the annihilation cross section.

## 1.2 Popular WIMP Candidates

For many reasons, we expect that there are new particles and forces (beyond the SM) near the electroweak scale. The strongest motivator is the electroweak hierarchy problem, which amounts to the fact that the scale of electroweak symmetry breaking appears to be unstable to quantum corrections. Proposals to solve this problem include supersymmetry (SUSY), extra dimensions, and new strong forces. The WIMP miracle described above gives a further piece of motivation for new physics at the electroweak scale. Indeed, many extensions of the SM contain (or can accommodate) a WIMP DM candidate.

The most popular extension of the SM is supersymmetry. Exact supersymmetry predicts that every SM particle should have a *superpartner* with the same mass and quantum numbers, but with a spin differing by half a unit. For example,

$$\begin{array}{ccc} \text{fermion} & f \leftrightarrow \tilde{f} & \text{sfermion} \\ (s=1/2) & & (s=0) \\ & & \\ \text{gauge boson} & A_\mu \leftrightarrow \tilde{A} & \text{gaugino} \\ (s=1') & & (s=1/2) \\ & & \\ \text{Higgs} & H \leftrightarrow \tilde{H} & \text{Higgsino} \\ (s=0) & & (s=1/2) \end{array} \tag{4}$$

The minimal supersymmetric extension of the SM (MSSM) has a superpartner for every SM particle, and basically nothing else. The lone exception is the Higgs sector, where two scalar $SU(2)_L$ Higgs doublets $H_{u,d}$ are required along with their Higgsinos $\tilde{H}_{u,d}$.

Supersymmetry stabilizes the electroweak scale by imposing a cancellation of quantum corrections to the Higgs fields (which induce electroweak symmetry breaking) between SM particles and their superpartners. The dangerous corrections cancel exactly if supersymmetry is an exact symmetry of Nature. However, this would also imply scalar electrons (selectrons) with the same mass as the electron, a possibility that is very firmly ruled out. On the other hand, if supersymmetry is broken the superpartners can be heavier than their SM counterparts. It turns out that broken supersymmetry can still protect the electroweak scale as long as all the operators that break SUSY have couplings of positive mass dimension that are not too large. This type of breaking is called *soft* because its effects become negligible at energies much larger than the scale of the SUSY-breaking couplings. By not too large, the quantitative requirement is $m_{soft} \lesssim 1000$ GeV, which implies that the superpartners must have masses close to this value. The LHC is currently probing this regime.

Even with soft breaking, the addition of superpartners to the SM can lead to all sorts of bad things happening (like rapid proton decay) unless we also impose a further symmetry called $R$-parity.[1] This is $\mathbb{Z}_2$ symmetry under which all the SM particles are even and all the superpartners are odd. As a result, superpartners must be created or destroyed in pairs, and the lightest superpartner (LSP) is stable.

[1] There are other possibilities as well, but $R$-parity is the simplest and most popular.

The LSP can be a viable DM candidate if it is stable and uncolored. In the MSSM, the two possibilities are the lightest neutralino and the lightest sneutrino. It turns out that the sneutrino, the scalar superpartner of a SM neutrino, is ruled out as a DM candidate due to limits on direct detection, so we will focus on a neutralino LSP. In all, there are four neutralinos $\chi_i^0$, $i = 1, 2, 3, 4$ in the MSSM with the labels such that the masses are increasing, $m_{\chi_1} \leq m_{\chi_2} \leq \ldots$. These four neutralinos are linear combinations of the Bino ($\tilde{B}^0$ = superpartner of the $U(1)_Y$ gauge boson), Wino ($\tilde{W}^3$ = superpartner of the neutral component of the $SU(2)_L$ gauge boson), and the Higgsinos ($\tilde{H}_u^0$, $\tilde{H}_d^0$ = superpartners of the neutral components of the two Higgs scalar doublets). Thus, we have

$$\chi_i^0 = N_{i1}\tilde{B}^0 + N_{i2}\tilde{W}^0 + N_{i3}\tilde{H}_d + N_{i4}\tilde{H}_u \ , \tag{5}$$

where $N_{ij}$ is a unitary mixing matrix. The mixing arises mostly from electroweak symmetry breaking. Note that the four neutralinos are all Majorana fermions, meaning that they are their own antiparticles.

In general, a mostly-Bino LSP annihilates inefficiently and produces too much DM through thermal freeze out, while a mostly Wino or Higgsino LSP produces too little. An acceptable thermal relic density can be obtained if the LSP is roughly equal mixture of the Bino and the other states [2]. There are also some special cases where the annihilation of a Bino-like neutralino is enhanced and the relic density comes out right. We will discusses some of the ways this can happen below.

Besides SUSY, there are a number of other theories of new physics that contain viable dark matter candidates. A specific example is Universal Extra Dimensions (UED), where the SM is able to propagate in a flat fifth dimension of length $R \sim \text{TeV}^{-1}$ bounded on either end by four-dimensional surfaces called *branes*. UED also has a reflection symmetry about the middle of the fifth dimension. The five-dimensional SM fields in this scenario can be expanded into towers of four-dimensional fields. The lightest particle in the tower is indentified with the SM state, while the heavier (bosonic) Kaluza-Klein (KK) modes have masses

$$m^2 = m_{SM}^2 + (n\pi/R)^2 \tag{6}$$

where $n \in \mathbb{Z}^{\geq}$ labels the KK level ($n = 0$ is the SM). The reflection symmetry means that there is a $\mathbb{Z}_2$ symmetry under which the KK modes transform as $(-1)^n$. As a result, the lightest KK mode (LKP) is stable and a candidate for the DM provided it is neutral.

## 1.3 Special Cases for DM Annihilation

As mentioned above, a mostly-Bino LSP tends to produce too much DM by thermal freeze-out. However, such a state can produce the correct amount of DM in certain special cases. We will describe two of the most popular examples here: resonant enhancement, and coannihilation.

Consider the annihilation channel $\chi_1^0\chi_1^0 \to h^0 \to b\bar{b}$ by way on intermediate $s$-channel Higgs boson. The squared amplitude for the annihilation cross contains a Higgs propagator, so it goes like

$$\mathcal{M} \propto \left| \frac{1}{s - m_h^2 + i m_h \Gamma_h} \right|^2 \tag{7}$$

where $s = (p_{\chi_1} + p_{\chi_2})^2 \simeq 4m_\chi^2$ and $\Gamma_h = \tau_h^{-1}$ is the decay width of the Higgs. Since $\Gamma_h \ll m_h$, the annihilation cross section will be strongly enhanced for $m_\chi \simeq m_h/2$ corresponding to the intermediate Higgs being nearly on-shell.

A second important special case is coannihilation. In writing the Boltzmann equation for the freeze out of $\chi$, we implicitly assumed that the only relevant particles were $\chi$ and those of the SM. In SUSY (and other scenarios), there may be other new particles that can also play an important role in determining the relic density. For example, $\chi_1^0$ particles can be created through the decay $\tilde{q} \to q\chi_1^0$ (where $\tilde{q}$ is a squark), and they can annihilate via $\tilde{q}\chi_1^0 \to q\,g$. Coannihilation refers to all these additional reactions that we have not yet considered.

In many situations, the coannihilation reactions are not important. For example, suppose the squark $\tilde{q}$ is much heavier than the $\chi_1^0$ LSP. In this case the squarks will freeze out on their own and decay down to $\chi_1^0$ well before $\chi_1^0$ freezes out. Since $\chi_1^0$ remains close to thermodynamic equilibrium before freeze out, the extra $\chi_1^0$ particles produced by these decays will annihilate away as this state tracks its equilibrium value.

The situation where coannihilation becomes important occurs when $\chi_1^0$ is close in mass (within about 20% or so) with another superpartner. In this case, it is no longer a good approximation to treat $\chi_1^0$ alone. Instead, we can track the total number density of all superpartners $n_{super} = \sum_{i=super} n_i$ since they will all eventually either annihilate to the SM or decay to $\chi_1^0$, so that $n_{super} = n_\chi$ at late times. Coannihilation can increase or decrease the relic density of $\chi$ compared to what it would obtain on its own. However, the effects tend to be greatest when there is a superpartner $\tilde{q}$ that is nearly degenerate with $\chi_1^0$ and has a much larger annihilation cross section. If so, the number density of $\chi_1^0$ can be depleted very efficiently by $\chi_1^0\chi_1^0 \to \tilde{q}\tilde{q}^*$ followed by $\tilde{q}\tilde{q}^* \to SM\,S\bar{M}$, instead of just $\chi_1^0\chi_1^0 \to SM\,S\bar{M}$.

Although we have only mentioned coannihilation in the context of SUSY, it can also play an important role in other theories containing DM. A specific example is minimal UED, where many of the KK partners are very close in mass and the LKP tends to be the KK photon [5]. The near-degeneracy of many states greatly reduces the final LKP density.

## 1.4 The WIMPless Miracle

The cross section in Eq. (1) goes like $(g_\chi^2/m_\chi)^2$. This gives about the right relic density for a WIMP, with $m_\chi \sim 100$ GeV and $g_\chi \sim 1$. However, we could also get about the right DM density from a particle that is much lighter but with much smaller couplings to the SM, such that the ratio $g_\chi^2/m_\chi$ is about the same as for a WIMP. This scenario feature is sometimes called the WIMPless miracle [6]. It can occur very naturally in gauge-mediated supersymmetry in the presence of a new force that couples only very weakly to the SM, since there $m_\chi \propto g_\chi^2$ so that the ratio remains constant.

# 2 Non-Thermal Dark Matter Creation

We have discussed thermal DM creation extensively. By thermal, we means specifically that the DM species was in thermodynamic equilibrium with the cosmological plasma before freezing out. There are also many non-thermal ways for the DM density to be created, and we will describe some of them here.

## 2.1 Gravitino Dark Matter

Supersymmetry as a global symmetry is an extension of the Poincaré symmetries of flat space. In the same way we can obtain general relativity (GR) by elevating the Poincaré symmetries to local coordinate transformations, we can extend global supersymmetry to a local symmetry called supergravity that extends GR [7]. In supergravity, the spin $s' = 2$ graviton obtains a $s' = 3/2$ superpartner called the gravitino, $\Psi_\mu$. The gravitino can be viable candidate for the DM, but only if it is produced non-thermally.

In the limit of exact supersymmetry, the gravitino is degenerate with the graviton and therefore massless. When supersymmetry is broken, the gravitino can acquire a non-zero mass. However, the massless $s' = 3/2$ has fewer physical polarizations than a massive $s = 3/2$ state. The additional degrees of freedom are acquired by eating the would-be massless *goldstino* $s = 1/2$ fermion, which is the supersymmetry analog of a Goldstone boson. This *super Higgs* mechanism is completely analagous to the regular Higgs mechanism where the gauge boson of a spontaneously broken gauge symmetry acquire a mass and a longitudinal component by eating the would-be Goldstone boson[7].

Supersymmetry breaking is thought to occur in a hidden sector (that does not couple directly to the MSSM), and communicated to the MSSM superpartners by messenger particles of mass $M_*$. The breakdown of SUSY is usually described by a non-zero value of the order parameter $F$, which has a mass dimension equal to two. In terms of these quantities, the scale of soft supersymmetry breaking in the MSSM $m_{soft}$ and the gravitino mass $m_{3/2}$ are given by [1]

$$m_{soft} = C_* \frac{F}{M_*}, \qquad m_{3/2} = \frac{F}{\sqrt{3} M_{\rm Pl}} \,. \tag{8}$$

The constant $C_*$ depends on the details of the messengers. Some examples are:

$$\begin{array}{lll} \text{gravity mediation} & C_* \sim 1 & M_* = M_{\rm Pl} \\ \text{gauge mediation} & C_* \sim \frac{g^2}{(4\pi)^2} & M_* \leq \frac{(4\pi)^2}{g^2} M_{\rm Pl} \\ \text{anomaly mediation} & C_* \sim \frac{g^2}{(4\pi)^2} & M_* = M_{\rm Pl} \end{array} \tag{9}$$

This implies that $m_{soft} \sim m_{3/2}$ in gravity mediation, $m_{soft} \gg m_{3/2}$ in gauge mediation, and $m_{soft} \ll m_{3/2}$ in anomaly mediation. Thus, in either gauge or gravity mediation, the gravitino can be the LSP and a candidate for the DM.

To estimate the relic abundance of a gravitino LSP, we need to know how it couples to the SM. The general form of the coupling is [1]

$$-\mathscr{L} \supset \frac{1}{M_{\rm Pl}}(\partial_\mu \tilde{f})\bar{f}\gamma^\mu\gamma^\nu\Psi_\nu + \frac{i}{8M_{\rm Pl}}\bar{\Psi}_\mu[\gamma^\nu,\gamma^\rho]\tilde{A}F_{\nu\rho} \ . \tag{10}$$

The coupling strength is evidently gravitational, although this is a bit misleading. The longitudinal parts of the gravitino are made up by the Goldstino $\tilde{\Psi}$ which couples according to $1/F$, and the enhanced coupling appears in gravitino polarization sum. For processes with characteristic energies $E \gg m_{3/2}$, this effect can be handled by making the substitution [1]

$$\Psi_\mu \to \sqrt{2/3}\,\partial_\mu\tilde{\Psi}/m_{3/2} \ . \tag{11}$$

Note that $m_{3/2}M_{\rm Pl} \sim F$.

The very weak couplings of gravitinos means that if it was ever in equilibrium with cosmological plasma, it will decouple from the plasma while it is still relativistic. Assuming no other production mode, this implies [8]

$$\Omega_\Psi h^2 \simeq (0.1)\left(\frac{m_{3/2}}{100\,\text{eV}}\right) \ . \tag{12}$$

Such a small mass corresponds to warm dark matter, and would lead to the relic gravitinos streaming out of overdense regions resulting in the washout of matter perturbations. This is ruled out unless $m_{3/2} \gtrsim 1$ keV (for $\Omega_\Psi h^2 = 0.1$ kept fixed) [9]. Thus, for the gravitino to make up the DM, it must be produced in some other way and be heavier than $m_{3/2} \simeq 1$ keV. Moreover, the mere existence of a stable gravitino LSP implies that the history of the Universe be such that it was never thermalized.[2]

Gravitino DM can work through several non-thermal mechanisms. These include decays of the MSSM superpartners to gravitinos, production by thermal scattering that is too weak to cause equilibration, and from the decays of very heavy and long-lived particles. We will describe the first two cases here, and leave the more general third case to the next subsection.

When the gravitino is the LSP, the lightest MSSM superpartner $\tilde{X}$ will be the next-to-LSP (NLSP). It will decay to the gravitino at the rate [8]

$$\Gamma(\tilde{X} \to X\Psi) \simeq \frac{1}{48\pi}\frac{m_{\tilde{X}}^5}{m_{3/2}^2 M_{\rm Pl}^2} \ . \tag{13}$$

The lighter the gravitino, the faster the decay. For $m_{\tilde{X}} \sim 100$ GeV, these decays will occur after $\tilde{X}$ undergoes thermal freeze out (assuming it had once equilibrated) for $m_{3/2} \gtrsim 100\,\text{keV}$. Since each NLSP decay produces one stable gravitino, this gives

$$\Delta\Omega_\psi h^2 = \left(\frac{m_{3/2}}{m_{\tilde{X}}}\right)\Omega_{\tilde{X}}h^2 \ , \tag{14}$$

[2]There is an exception for very light gravitinos, $m_{3/2} \lesssim 10\,\text{eV}$, since these will be sufficiently dilute to avoid messing the matter power spectrum [9].

where $\Omega_{\tilde{X}}h^2$ is the relic density $\tilde{X}$ would produce if it were stable. When the DM comes primarily from this mechanism, it is called the *SuperWIMP* scenario [10]. For many NLSP varieties, it is very strongly constrained (or ruled out) by the fact that the NLSP decay occurs during primordial nucleosynthesis; the energetic decay products can destroy the light elements that have been created, altering their abundances. When $m_{3/2} < 100$ keV, it is produced copiously by $\tilde{X}$ decays that occur before freeze out [8]. This creates too much dark matter unless the MSSM itself was never thermalized, which can be arranged if the temperature of the Universe never exceeded $T_{RH} \simeq 100$ GeV while it was radiation-dominated.

Gravitinos can also be produced by thermal scattering, even if they never quite attain thermodynamic equilibrium. The relevant Bolztmann equation for this is identical to the one for thermal freeze. The domiant contribution to gravitino production comes from gluino processes such as $\tilde{g}g \to \tilde{\Psi}g$ and $gg \to \tilde{g}\tilde{\Psi}$. The net contribution is

$$\Delta\Omega_{\Psi}h^2 \simeq (0.1)\left(\frac{100\text{ keV}}{m_{3/2}}\right)\left(\frac{\text{TeV}}{M_3}\right)\left(\frac{T_{RH}}{2\text{ TeV}}\right) , \tag{15}$$

where $M_3$ is the mass of the gluino and $T_{RH}$ is again the maximal temperature the Universe attained while it was dominated by radiation.

In Fig. 1 we summarize the upper limit on the reheating temperature assuming a gravitino LSP and an MSSM NLSP with mass close to $m_{\tilde{X}} \sim 100$ GeV. This plot is somewhat out of date for several reasons. First, it only applies the condition $\Omega_{\Psi}h^2 < 1$. Thus, in the right region ($m_{3/2} > 100$ keV) the current limit on $T_{RH}$ is about an order of magnitude stronger. For $m_{3/2} > 10$ GeV, there are additional limits from NLSP decays to gravitinos during nucleosynthesis. For $10\,\text{eV} < m_{3/2} < 1$ keV, there may be additional exclusions from the washout of large-scale structure.

## 2.2 Massive Particle Decays

Another to produce DM non-thermally is through the decays of a long-lived, massive particle. Consider a heavy particle $P$ with mass $m_P$ much larger than the DM mass $m_\chi$. If $P$ interacts very weakly with the SM (or any other light states), it can freeze out with a very large initial abundance and decay at a much lighter time to the SM and DM. If the lifetime is very long and it becomes non-relativistic, the energy density in the abundance of $P$ particles can come to dominate the Universe. In this case, the Universe can go from radiation domination to matter domination (by $P$), and later return to radiation domination when the decays of $P$ particles create energetic SM states. This process is called *reheating*. DM can also be created during reheating, and its density will be very different from what it would obtain by thermal freeze out if the reheating temperature is far below its freeze-out temperature.

To describe this process qualitatively, let us assume that $P$ decays to radiation and to dark matter with relative partial decay widths $\epsilon\Gamma_P$ and $(1-\epsilon)\Gamma_P$ for some $\epsilon \lll 1$. This will inject more radiation and some dark matter into the Universe. These processes are described

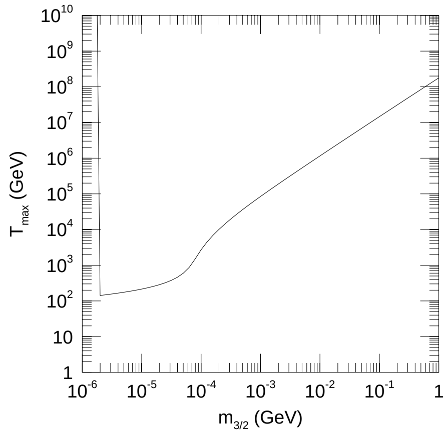


Figure 1: Upper limit on $T_{RH}$ as a function of $m_{3/2}$ if there is gravitino LSP. Figure taken from Ref. [11].

by the differential equations [12][3]

$$\begin{aligned}
\frac{d\rho_P}{dt} + 3H\rho_P &= -\Gamma_P \rho_p & (16)\\
\frac{d\rho_R}{dt} + 4H\rho_R &= +(1-\epsilon)\Gamma_P \rho_P & (17)\\
\frac{dn_\chi}{dt} + 3Hn_\chi &= +\epsilon\Gamma_P(\rho_P/m_P) - \langle\sigma v\rangle\left(n_\chi^2 - n_{\chi_{eq}}^2\right) & (18)\\
H = \left(\frac{\dot{a}}{a}\right) &= \sqrt{\frac{8\pi G}{3}}(\rho_P + \rho_R + \rho_\chi)^{1/2} & (19)
\end{aligned}$$

The net result of these equations is that the temperature at which radiation takes over again after the $P$ domination is

$$T_{RH} \simeq g_*^{-1/4}\sqrt{(1-\epsilon)M_{\rm Pl}\Gamma_P}\ . \tag{20}$$

This occurs at time $t \simeq \tau_P = \Gamma_P^{-1}$.

For the resulting DM density, the result depends on $T_{RH}$ relative to thermal freeze out. For $T_{RH} \ll T_f$, the annihilation term in Eq. (18) can be neglected and the resulting yield is

$$Y_\chi \simeq \epsilon\left(\frac{T_{RH}}{m_P}\right)\ . \tag{21}$$

[3]These equations assume $g_*$ remains constant throughout the process. A more general treatment can be found in Ch.5.3 of Ref. [12].

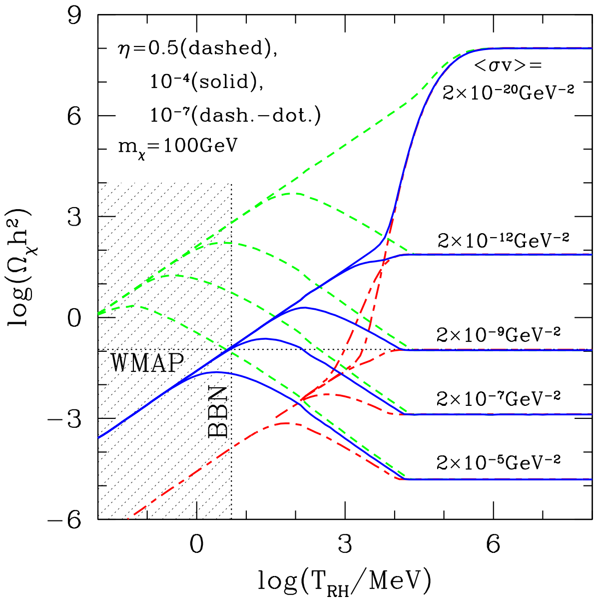


Figure 2: Relic densities of $\chi$ including production from the decays of a heavy particle $P$. The results are shown for various values of the annihilation cross section and $\eta = \epsilon(100 \text{ TeV}/m_P)$. Figure taken from Ref. [13].

On the other hand, if $T_{RH} \gg T_f$ the DM particles will likely have enough time to re-equilibrate with the SM plasma, and their final relic density will be thermal. The general result for the intermediate case interpolates between these two limits [13]. We show their result in Fig. 2, where $\eta = \epsilon(100 \text{ TeV}/m_P)$.

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