Tutorial Ideas for # 4

1. **General Goldstinos**
   The existence of a massless goldstino can be shown on general grounds for a theory consisting of chiral and vector superfields. Consider such a theory with an Abelian vector multiplet (λ, A^μ, D) and a set of n chiral superfields Φ_i with gauge charges q_i.

   a) A general $U(1)$ transformation takes
   $$\Phi_i \;\to\; e^{iq_i\alpha}\Phi_i \;=\; (1 + iq_i\alpha + \ldots)\Phi_i \;,$$
   where we have specialized to infinitesimal transformations in the second equality. By expanding the superpotential to linear order in small α, find a necessary (and sufficient) condition for the superpotential to be gauge-invariant.

   b) Write down the full scalar potential of the theory and derive a general expression for the minimization conditions $\partial V/\partial\phi_i = 0$.

   c) Find the general fermion mass matrix in the basis (λ, ψ_i) assuming some of the scalar fields get VEVs $\langle\phi_i\rangle$.
   Hint: consider the mixed gauge terms and the superpotential bits, and keep in mind that in the given basis, the blocks of the matrix will have dimension 1×1, $1\times n$, $n\times 1$, *and* $n\times n$, *where* n *is the number of chiral superfields.*

   d) Define a $(1+n)$-dimensional column vector v_a by
   $$v_a = \left(\begin{array}{c} \langle D\rangle/\sqrt{2} \\ \langle F_i\rangle \end{array}\right) \;,$$
   where $\langle D\rangle = \langle g\sum_i q_i|\phi_i|^2\rangle$ and $F_i = \langle\partial W^\dagger/\partial\phi_i^\dagger\rangle$. Show that v_a is non-trivial if and only if there is supersymmetry breaking. When supersymmetry breaking is present, prove that v_a is a zero eigenvector of the mass matrix $\mathcal{M}_{ab}$. The corresponding massless fermion is the goldstino.
   Hint: use the results of parts a) and b).

2. **Gravitino Dynamics**
   These can have interesting effects in colliders and cosmology.

   a) Use dimensional analysis to estimate the decay rate of the gravitino to a particle-superpartner pair via the gravitino couplings of Eq. (106).

   b) Do the same for the decay $\widetilde{X} \to X + \widetilde{G}$ based on the couplings of Eq. (106) in `notes-02`. Comparing to Eq. (108), you should get a parametrically wrong answer when $m_{3/2} \ll m_{\widetilde{X}}$.

   c) The analysis of b) is wrong because the longitudinal goldstino enhancement was not taken into account. Repeat the estimate, but now use Eq. (107) to model the enhancement if applicable.

d) In gauge mediation, the gravitino is typically the LSP and the lightest MSSM superpartner NLSP can decay to it. Assuming the NLSP is a mostly-Bino neutralino with a mass of 100 GeV that decays primarily through $\chi_1^0 \to \gamma\widetilde{G}$, estimate the values of F and M_* such that the decay length $c\tau$ is 1 mm and 10 m. The first distance corresponds to the resolution of the LHC to detect delayed decays, and the second is the size of the LHC detector (so that $c\tau > 10$ m means that the NLSP decay is not seen in the detector).

3. **Supersymmetric Nonrenormalization**

The cancellations among loops of particles and superpartners have been formalized in a set of non-renormalization theorems. These are usually expressed in terms of an effective Lagrangian obtained by integrating out heavy physics down to the scale μ. When calculating with such an effective Lagrangian, only (Euclidean) momenta below $q_E^2 \leq \mu^2$ are to be considered in loops. The key non-renormalization result for chiral superfields is that the superpotential is not renormalized for a suitable choice of field coordinates. We will illustrate this for the Wess-Zumino model based on the chiral superfield Φ and the tree-level superpotential

$$W_0 = \frac{1}{2}m_0\Phi^2 + \frac{\lambda_0}{3!}\Phi^3 \ .$$

If supersymmetry is to be preserved at the quantum level, the interactions in the effective Lagrangian must be derivable from an effective superpotential. For an appropriate choice of field basis, this superpotential W_{eff} can depend only Φ, λ_0, m_0, and μ, and does not have to be renormalizable. Our goal will be to deduce its form.

a) Show that if we treat m_0 and λ_0 as superfields, the tree-level superpotential has a $U(1)_x$ symmetry under which $[\Phi]_x = 1$, $[m_0]_x = -2$, and $[\lambda_0]_x = -3$. Show also that the superpotential has a $U(1)_R$ symmetry for the charges $[\Phi]_R = 1$, $[m_0]_R = 0$, $[\lambda]_R = -1$.

b) What combinations of the integers (n_λ, n_m, n_Φ) produce monomials $\lambda_0^{n_\lambda} m_0^{n_m} \Phi^{n_\Phi}$ that are neutral under both $U(1)_x$ and $U(1)_R$? Show that the most general form is $(\lambda_0^a \Phi^b m_0^c)^n$ for some minimal integers (a, b, c) and any n.
$(\lambda_0\Phi/m_0)^n$ *for any* n

c) The quantum-corrected superpotential should respect these symmetries as well. Use this requirements and your previous result to show that the most general effective superpotential is

$$W_{eff} = m_0\Phi^2 \sum_{n\in\mathbb{Z}} a_n(\lambda_0^a\Phi^b m_0^c)^n$$

d) This effective superpotential should have smooth limits as $m_0 \to 0$ and $\lambda_0 \to 0$, and reproduce the tree-level result as $\lambda_0 \to 0$. What does this imply for the coefficients a_n?

e) What does this imply for the renormalization of W? Note that there will also be a wavefunction renormalization that rescales the kinetic terms.