Notes #3: Strong Coupling and Compositeness

David Morrissey
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A second approach to the hierarchy problem is to remove the Higgs as fundamental scalar. This can be done be replacing the fundamental Higgs scalar with a composite scalar bound state held together by a new strongly-interacting force. A related possibility, that has been mostly ruled out by the discovery of a SM-like Higgs boson, is that the new strong dynamics itself induces electroweak symmetry breaking. We will go over some of these ideas in these notes. Before discussing new forces, however, we will review some aspects of QCD and its relationship with electroweak symmetry breaking. This will serve as useful reference for the material to follow.

1 Aspects of QCD

Quantum Chromodynamics (QCD) is the accepted theory of the strong force. It is an $SU(3)$ gauge theory with matter quark fields transforming under the fundamental **3** irrep of the gauge group. The fundamental Lagrangian is:

$$\mathscr{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \sum_I \bar{q}_I(i\gamma^\mu D_\mu - m_I)q_I, \tag{1}$$

where $I = u,\, d,\, s,\, c,\, b,\, t$, and

$$D_\mu = \partial_\mu + ig_s t^a_3 G^a_\mu, \tag{2}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc}G^b_\mu G^c_\nu\ . \tag{3}$$

The matrices t^a_3 are of course the (eight) generators of the **3** of $SU(3)$. Gauge charges are called *colour* while the different species of 4-component Dirac fermion quarks are called *flavours*. The masses m_I of the different quark flavours are approximately

$$\begin{array}{lll} m_u \simeq 2.5\text{ MeV}, & m_d \simeq 5.3\text{ MeV}, & m_s \simeq 110\text{ MeV}, \\ m_c \simeq 1.25\text{ GeV} & m_b \simeq 4.5\text{ GeV}, & m_t \simeq 173\text{ GeV}. \end{array} \tag{4}$$

Of course, this structure fits in nicely with the rest of the SM, with the quark masses arising from electroweak symmetry breaking.

While the underlying QCD Lagrangian is very simple, the resulting dynamics are anything but. We never actually observe quarks or gluons as free asymptotic particles. Instead, at low energies (or long distances) we only ever see colour-neutral bound states of quarks and gluons. This stands in stark contrast to QED, where we certainly do see free particles charged under the gauge group – electrons for example. The absence of free colour-charged objects is called *confinement*.

Confinement is still not completely understood at the quantitative level. A large part of the reason for this is the breakdown of perturbation theory in QCD at low energies. Despite these challenges, it is still possible to construct a useful low-energy EFT for the bound states resulting from QCD confinement. Collectively these bound states are called *hadrons*, and the most important examples are *mesons* and *baryons*. The quantum numbers of these states can be matched to the colour-neutral quark operators

$$M \sim \bar{q}^i q'_j \delta^i_j, \qquad B \sim q_i q'_j q''_k \epsilon^{ijk}, \tag{5}$$

where i and j are colour indices.

1.1 Running Couplings at High Energy

A very rough idea of where confinement comes from can be obtained by examining the scale dependence of the renormalized QCD coupling $g_s(\mu)$. Here, μ corresponds to the scale at which the coupling is renormalized. When $\mu \sim p$, the value of this coupling coincides reasonably well with the physical QCD coupling strength in a process occurring at the characteristic momentum scale p. In a generic gauge theory, the running coupling $g(\mu)$ can be obtained by measuring the coupling at one momentum scale and solving the renormalization group (RG) equation to extrapolate it to other momentum scales. At one-loop order, the RG equation is [1]

$$\frac{dg}{dt} := \beta(t) = -\frac{b}{(4\pi)^3} g^3 \tag{6}$$

where the coefficient b is given by

$$b = \frac{11}{3} C_2(A) - \sum_r \frac{2}{3} T_2(r) - \sum_{r'} \frac{1}{3} T_2(r'), \tag{7}$$

where $C_2(A)$ is the Casimir of the adjoint (equal to $N^2 - 1$ for $SU(N)$), $T_2(r)$ is the trace invariant of the representation r (equal to 1/2 for a fundamental of $SU(N)$), $t = \ln(\mu/\mu_0)$, the first sum runs over all light 2-component fermion reps in the theory, and the second sum runs over all light complex scalar reps. By "light", we mean all reps with mass $m < \mu$. As μ falls below the mass of a particle in the (effective) theory, we implicitly remove it from the EFT so that it no longer contributes to the RG running. The leading-order matching condition for the running gauge coupling at the mass threshold $\mu = M$ is simply

$$\lim_{\mu \to M_-} g(\mu) = \lim_{\mu \to M_+} g(\mu). \tag{8}$$

That is, the running coupling is continuous across the mass threshold.

In QCD, for $\mu > m_t$ we have

$$b_{QCD} = \frac{11}{3} \times 3 \quad - \quad \frac{2}{3} \times \frac{1}{2} \times 2 \times 6 = 7. \tag{9}$$

In the second term, the 1/2 comes from $T_2(\mathbf{3}) = 1/2$, the 2 comes from the left and right 2-component parts of each quark, and the 6 comes from the six quark flavours. At lower energies, the RG beta-function coefficient becomes

$$b_{QCD} = \begin{cases} 7 & \mu > m_t \\ 23/3 & m_b < \mu < m_t \\ 25/3 & m_c < \mu < m_b \\ \dots \end{cases} \tag{10}$$

We can now compute the value of $g_s(\mu)$ at other values of μ after inputting the measured boundary value of the coupling,

$$\alpha_s(m_Z) \equiv \frac{g_s^2(\mu)}{4\pi} \sim 0.118, \tag{11}$$

into to the solution of Eq. (6).

Note that the sign of the RG coefficient b_{QCD} is such that the strong coupling becomes *weaker* at high energies. This property is called *asymptotic freedom.*[1] The flip side of this is that the QCD coupling grows large at low-energies. We can use this property to derive a dimensionful scale from the dimensionless QCD coupling. The solution to the RG equation (between thresholds) is

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + \frac{b_{QCD}}{2\pi} \ln\left(\frac{\mu}{\mu_0}\right). \tag{12}$$

With this in hand, it makes sense to define the QCD scale Λ_{QCD} as the point where $\alpha_s^{-1}(\mu)$ vanishes. This yields

$$\alpha_s(\mu) = \frac{2\pi}{b_{QCD} \ln(\mu/\Lambda_{QCD})} . \tag{13}$$

The appearance of a dimensionful scale from a dimensionless (but scale-dependent) coupling is called *dimensional transmutation*. Numerically, $\Lambda_{QCD} \simeq 200$ MeV, and this value characterizes the onset of strong coupling in QCD. In practice, QCD becomes strongly-coupled a little earlier than this, near $E \sim 1$ GeV, which is roughly the mass scale of the light baryons.

1.2 QCD at Low Energies

At low energies, $E \lesssim 1$ GeV, the QCD degrees of freedom one observes are mainly baryons and mesons. It is therefore much more efficient to describe this system with a field theory that treats these as the dynamical fields instead of using the seemingly more fundamental quarks and gluons. In other words, we want the low-energy EFT of QCD.

This isn't so easy to come by. Quarks and gluons are weakly-coupled at energies well-above Λ_{QCD}, and baryons and mesons are weakly-coupled at energies much below it, but there is strong coupling in between where we would like to match them up. Without perturbation theory, one must address the full dynamics of the theory. This is done numerically using

[1] This property is a very special feature of non-Abelian gauge theories.

lattice field theory to simulate the strongly-coupled dynamics of gluons and quarks [2, 3]. A second approach, the one we will discuss here, is to simply write an effective low-energy theory with the appropriate set of degrees of freedom and all possible interactions consistent with the underlying symmetries [4, 5, 6]. The coefficients of these interactions can be set by comparing to observation, or by computing them from QCD using lattice simulations.

To derive an EFT for low-energy QCD, a very helpful observation is that the u and d quarks are both very light relative to Λ_{QCD}, the s quark is somewhat light, and the other quarks are relatively heavy. Thus, to study the lightest QCD degrees of freedom we should be able to integrate out the c, b, and t quarks and work only with the u, d, and s quarks. To simplify the discussion here, let's also ignore the s quark for now and treat the u and d quark masses as small corrections that we will handle perturbatively.

With only the u and d quarks, and neglecting their very small masses (relative to $\Lambda_{QCD} \sim$ 200 MeV), the QCD Lagrangian of Eq. (1) has a global $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$ flavour symmetry under which the fields transform as

$$q_{L\,I} \to e^{i\alpha_V} e^{i\alpha_A} L_{IJ} q_{L\,J}, \qquad q_{R\,I} \to e^{i\alpha_V} e^{-i\alpha_A} R_{IJ} q_{R\,J}, \tag{14}$$

where $q = (u,\, d)^t$, $I,\, J = u,\, d$, and L and R are $SU(2)$ transformations for the fundamental rep in flavour space. Of the factors making up this global symmetry group, only the $U(1)_A$ part is anomalous with respect to QCD ($SU(3)_c$), meaning that the remaining factors are all good symmetries at the quantum level. Therefore we should try to build a low-energy effective theory that is symmetric under $G_{flav} = SU(2)_L \times SU(2)_R \times U(1)_V$.

Before attempting to write down such a theory, let us mention one additional and essential fact: strong coupling in QCD generates an expectation value for the gauge-invariant $\bar{q}q$ quark operator

$$\langle \bar{q}_{R\,J} q_{L\,I} \rangle \;=\; \Lambda_{QCD}^3 \delta_{IJ}, \tag{15}$$

where I and J run over u, d, and s. This quark condensate expectation value *does not* respect the full (non-anomalous) global symmetry group. Applying a general G_{flav} transformation to this operator, the expectation value is not invariant and changes into

$$\Lambda_{QCD}^3 \delta_{IJ} \to \Lambda_{QCD}^3 (LR^\dagger)_{IJ}. \tag{16}$$

Thus, the $\bar{q}q$ expectation value spontaneously breaks G_{flav} to a smaller subgroup. It is not hard to see that this subgroup is $H_{flav} = SU(2)_V \times U(1)_V$, where $SU(2)_V$ is the subgroup of $SU(2)_L \times SU(2)_R$ transformations with $L = R$. The global G_{flav} symmetry is sometimes called a *chiral* symmetry, and its breaking is referred to as *chiral symmetry breaking.*

This spontaneous breakdown of $G_{flav} \to H_{flav}$ has three broken generators, and we expect three corresponding massless Nambu-Goldstone bosons (NGBs). Since the other QCD degrees of freedom are generically expected to pick up masses on the order of Λ_{QCD}, it makes sense to build a low-energy EFT with only these NGBs as the light degrees of freedom. Once we do, we will try to identify these light NGB fields with observed particles. The unbroken $SU(2)_V$ symmetry is called *isospin*, while the unbroken $U(1)_V$ corresponds to baryon number (up to an overall normalization of the generators). Since chiral symmetry breaking plays an essential role in constructing this EFT, it is usually called *chiral perturbation theory.*

There isn't a unique way to build the EFT for the NGBs, but we should at least make sure the EFT respects the full underlying G_{flav} global symmetry, has three explicit degrees of freedom, and that the corresponding field excitations vanish in the vacuum configuration of the theory. A convenient way to accomplish these tasks is to use field variables that look like spacetime-dependent G_{flav} transformations acting on the vaccuum. Here, this corresponds to building the theory out of the 2×2 matrix of fields

$$\Sigma(x) = \exp\left[2i\Pi^a(x)t^a/f\right], \tag{17}$$

where $t^a = \sigma^a/2$, the $\Pi^a(x)$ are the dynamical fields, and f is an as-yet unspecified parameter with dimension of mass. Under G_{flav} transformations, this field matrix is assumed to transform as

$$\Sigma(x) \to L\,\Sigma(x)\,R^\dagger. \tag{18}$$

Thus, $\Sigma(x)$ transforms in the same way as the quark condensate, Eqs. (15,16).

It is instructive to look at how the Π fields transform under the action of G_{flav} and H_{flav}. For this it is useful to write (without loss of generality)

$$L = e^{ic_A^a t^a} e^{ic_V^b t^b}, \qquad R = e^{-ic_A^a t^a} e^{ic_V^b t^b}. \tag{19}$$

In this form we see that $SU(2)_V$ coincides with $c_A^a = 0$. Acting with an infinitesimal $SU(2)_V$ transformation on Σ ($L = R \equiv V$) we find that

$$\Sigma(x) \to V\Sigma V^\dagger = \exp\left[2i\,V\Pi^a(x)t^a V^\dagger/f\right], \tag{20}$$

and that

$$\Pi^a \to \Pi'^a = \left(\delta^{ac} - f^{abc}c_V^b\right)\Pi^c + \mathcal{O}(c_V^2). \tag{21}$$

Thus Π transforms linearly and in the adjoint representation of $SU(2)_V$. Under transformations by the broken generators (*i.e.* $c_V = 0$), we find that

$$\Pi \to \Pi' = \Pi + f c_A^a t^a + \mathcal{O}(c_A^2). \tag{22}$$

This is a non-linear transformation on Π, and it takes precisely the shift form we expect for a Goldstone boson field. These nice transformation properties are the reason why the seemingly funny choice of field variables made in Eq. (18) is so useful.

We can now write down a Lagrangian in terms of the field variables Σ. Even though part of G_{flav} is spontaneously broken, the low-energy effective Lagrangian should still be symmetric under the full group. Looking simplest real and symmetric combination of Σ fields is $\Sigma^\dagger\Sigma = \mathbb{I} = \Sigma\Sigma^\dagger$, but this is trivial. To get something non-trivial, we need derivatives. The lowest-order term is

$$\partial_\mu\Sigma^\dagger\partial^\mu\Sigma \to R(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma)R^\dagger \tag{23}$$

implying that the trace $tr(\partial\Sigma^\dagger\cdot\partial\Sigma)$ is real and invariant under the symmetries. Thus, the lowest-order term one can write within this theory is

$$\begin{aligned}\mathscr{L}_{p^2} &= \frac{f^2}{4}\,tr(\partial\Sigma^\dagger\cdot\partial\Sigma) \\ &= \frac{1}{2}(\partial\Pi^a)^2+\frac{1}{3f^2}tr[(\partial\Pi\cdot\Pi)^2]+\ldots\end{aligned}\tag{24}$$

The first term is a canonical kinetic term for the Π fields while the second is an interaction term. Relative to the first term, the leading interaction is suppressed by a factor of p^2/f^2. Higher-order terms in the expansion of this operator are suppressed by additional powers of p^2/f^2. Thus, this theory is only useful as an EFT valid for $p^2 \ll f^2$.

The next set of terms come with suppressions of at least p^4/f^4. They are

$$\begin{aligned}\mathscr{L}_{p^4} &= L_1\left[tr(\partial\Sigma^\dagger\cdot\partial\Sigma)\right]^2 \\ &\quad + L_2\,tr(\partial_\mu\Sigma^\dagger\partial_\nu\Sigma)\,tr(\partial^\mu\Sigma\partial^\nu\Sigma) \\ &\quad + L_3\,tr(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma\,\partial_\nu\Sigma^\dagger\partial^\nu\Sigma),\end{aligned}\tag{25}$$

where L_1, L_2, and L_3 are unknown dimensionless coupling constants. Note that all these terms involve only derivatives of Σ, which is required by the non-linear transformation properties of this field under transformations induced by the spontaneusly broken generators. There is an infinite set of even higher-order terms that can be added. In practice, however, as long as $p^2 \ll f^2$ and we require a finite level of accuracy in our theoretical predictions, only a finite set of operators need be considered.

With a sensible EFT in hand, the next step is to connect the dynamical fields it contains to physical particles and to fix the numerical value of f (and the other couplings). We can identify the electromagnetic charges of the Π^a fields by noting that a subgroup of $SU(2)_V \times U(1)_V$ coincides with (spacetime independent) QED gauge transformations. The corresponding generator is[2]

$$Q \equiv t_L^3 + t_R^3 + \frac{1}{6}\mathbb{I}.\tag{26}$$

Applying such a transformation to Π, we find that its components have electric charges $Q = 0,\, \pm 1$, with

$$\pi^0 = \Pi^3, \qquad \pi^\pm = \frac{1}{\sqrt{2}}(\Pi^1 \mp i\Pi^2).\tag{27}$$

It is natural to identify these states with the lightest strongly-interacting colour-singlet paticles: the neutral and charged pions. Furthermore, the pions are known to be pseudoscalars, which is also true for NGBs. At this point the pions in our EFT are exactly massless whereas the real pions have masses of about 135 MeV. We will see shortly how to account for this apparent discrepancy.

[2]We are treating QED as a small perturbation on QCD here, and it should be clear that this exact gauge symmetry explicitly breaks the global flavour symmetries we are discussing by a small amount.

We would also like to fix the dimensionful parameter f in our theory. For this, we can match the conserved current operators in both the underlying QCD theory of quarks and gluons with the current operators in the pionic EFT. In the quark theory we have

$$\begin{aligned} j_V^\mu &= \bar{q}\gamma^\mu q, & j_A^\mu &= \bar{q}\gamma^\mu\gamma^5 q \\ j_L^{a\,\mu} &= \bar{q}\gamma^\mu P_L t^a q, & j_R^{a\,\mu} &= \bar{q}\gamma^\mu P_R t^a q. \end{aligned} \tag{28}$$

In the EFT, we find using Noether's theorem

$$\begin{aligned} j_V^\mu &= i\left(\pi^+\partial^\mu\pi^- - \pi^-\partial^\mu\pi^+\right) + \ldots \\ j_L^{a\,\mu} &= -i\frac{f^2}{2}tr(\Sigma^\dagger t^a\partial^\mu\Sigma) = f\,tr(t^a\partial^\mu\Pi) + \mathcal{O}(\Pi^2), \\ j_R^{a\,\mu} &= -i\frac{f^2}{2}tr(\Sigma\, t^a\partial^\mu\Sigma^\dagger) = -f\,tr(t^a\partial^\mu\,\Pi) + \mathcal{O}(\Pi^2). \end{aligned} \tag{29}$$

Consider now the decay of a negatively charged pion. This proceeds through a W^-, and its amplitude is proportional to the matrix element

$$\langle\mu^-\bar{\nu}_\mu\,|H_{int}|\,\pi^-(p)\rangle \tag{30}$$

with the interaction operator given by

$$H_{int} = \frac{g^2}{2m_W^2}(\bar{u}\gamma^\mu P_L d)\,(\bar{\mu}\gamma_\mu P_L\nu_\mu). \tag{31}$$

Contracting fields with external states, the matrix element factorizes into a simple leptonic piece, and a complicated hadronic piece given by

$$\langle 0\,|\bar{u}\gamma^\mu P_L d\,|\pi^+(p)\rangle \equiv i\frac{1}{\sqrt{2}}\,f_\pi p^\mu, \tag{32}$$

where the right-hand side is fixed by Lorentz invariance. Now, we can write this quark operator in terms of a current

$$\bar{u}\gamma^\mu P_L d = \left(j_L^{1\,\mu} + j_L^{2\,\mu}\right) = \frac{1}{\sqrt{2}}\,f\,\partial^\mu\pi^- + \mathcal{O}(\pi^2). \tag{33}$$

Plunking this into the pion matrix element, we see that to leading order

$$f = f_\pi \simeq 93 \text{ MeV}, \tag{34}$$

where the latter numerical value is extracted from the measured the rate of pion decays. Measurements of pion scattering can be used in a similar way to fix the values of L_1, L_2, and L_3.

The last piece of the puzzle is explaining the pion masses. Recall that in setting up our EFT for pions, we purposely ignored the small u and d quark masses. These can be put back in and treated as small perturbations since $m_{u,d} \ll f$. Even though they are small, these masses play an essential role because they *explicitly* break $SU(2)_L \times SU(2)_R$ down to $SU(2)_V$ for $m_u = m_d$, and down to nothing at all for $m_u \neq m_d$ (which seems to be the case). As a result of this explicit breaking, our $SU(2)_L \times SU(2)_R$ global symmetry is only approximate, with symmetry breaking effects on the order of $m_{u,d}/f$. The would-be NGB pions are now only pseudo-NGBs that have small but non-zero masses proportional to m_u and m_d.

We will write the quark mass matrix as $M = diag(m_u, m_d)$, so that

$$-\mathscr{L} \supset \bar{q}_R M q_L + \bar{q}_L M^\dagger q_R. \tag{35}$$

If this fixed matrix did transform along with the quark fields under G_{flav} according to

$$M \to RML^\dagger, \tag{36}$$

we would regain the full G_{flav} invariance. Of course it doesn't, but if we pretend it does and impose G_{flav} symmetry with this imagined transformation law, we can keep track of the symmetry breaking effects in an organized way. The leading EFT term that can be written with this in mind is

$$-\mathscr{L} \supset \frac{1}{2}\tilde{\Lambda}^3\, tr(M\Sigma) + h.c. \tag{37}$$

where we expect $\tilde{\Lambda} \sim \Lambda_{QCD}$. Expanding this out, we find a pion mass term of

$$m_\pi^2 f_\pi^2 = \tilde{\Lambda}^3(m_u + m_d). \tag{38}$$

As expected, the pion masses go to zero as the underlying quark masses vanish.

So far we have neglected the strange quark, but it turns out to be a pretty good approximation to include it as well, treating its mass as another small perturbation. The resulting theory now has an approximate $SU(3)_L \times SU(3)_R \times U(1)_V$ global symmetry that is spontaneously broken by the QCD vacuum down to $SU(3)_V \times U(1)_V$. This produces an octet of eight (pseudo-) NGBs that can be identified with the pions and kaons. More precisely, the components of Σ now correspond to

$$\Pi^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}. \tag{39}$$

As before, we can derive the approximate masses of these states by adding the 3×3 mass matrix $M = diag(m_u, m_d, m_s)$ to the theory as a small perturbation. These masses agree pretty well with the observed values. A different set of technology is needed to describe mesons involving c and b quarks. The top quark, being very heavy, decays too quickly to form meson bound states.

Low-energy QCD also involves baryons and more complicated meson excitations. These cannot be identified with NGB modes, and are somewhat harder to describe in an EFT. The presence of an approximate global $SU(3)_V \times U(1)_V$ still turns out to be very useful in

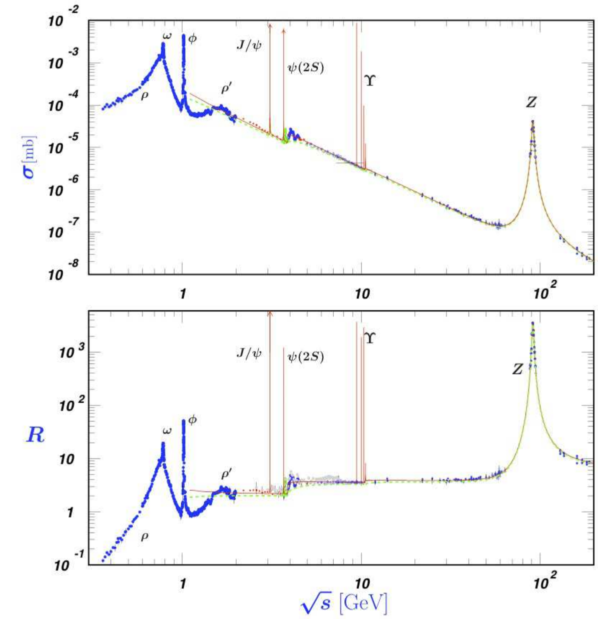

Figure 1: Total cross section and ratio $R(s)$ for inclusive hadronic production in e^+e^- collisions as functions of the CM energy $\sqrt{s}$.

organizing the various baryon states, and the lowest-lying modes naturally fill out singlet and octet representations. These symmetries also constrain the form of interactions between baryons and mesons, and symmetry breaking effects can again by added perturbatively.

Before finishing up, let us mention a couple of additional points. First, we have not looked into the effects of electromagnetism on the chiral perturbation theory EFT we have discussed. Relative to the dominant QCD dynamics, we can treat QED effects as small perturbations to the leading behaviour we have discussed here. In a few cases, however, QED effects can be very important. In particular, the G_{flav} global symmetry we have discussed is broken explicitly by QED (*e.g.* the u and d quarks transforming as $SU(2)_{L,R}$ doublets have different QED charges), and it also has an anomaly with respect to QED. The explicit breaking leads to electromagnetic contributions to the pion masses that split the values of the charged and neutral states. The anomaly in G_{flav} relative to QED leads to a coupling between the π^0 and two photons. It turns out that this anomaly-induced coupling leads to the dominant decay channel of the neutral pion: $\pi^0 \to \gamma\gamma$ has a branching fraction of nearly 99%.

Second, going beyond chiral perturbation theory, there are also many heavier QCD excitations. Even though it is very difficult to predict their masses, we do expect them to respect the approximate G_{flav} global symmetry and to appear in complete representations of the unbroken H_{flav} subgroup. The neutral components are often identified experimentally as resonances in $e^+e^- \to$ hadrons. This can been seen in Fig. 1, where we show the measured hadronic production cross section and the ratio $R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$.

2 Technicolor

Strongly-coupled theories of electroweak symmetry breaking attempt to achieve this breaking in much the same way as chiral symmetry breaking in QCD. In fact, if the SM did not have a Higgs boson, EWSB would still occur through QCD effects, although at energies near Λ_{QCD}. Technicolor theories are extensions of the SM where an analagous mechanism of EWSB is realized by adding a new *technicolor* gauge force that becomes strong near 100 GeV (rather than $\Lambda_{QCD} \sim 100$ MeV). In this section we will show how QCD would lead to EWSB in the absence of a Higgs, and then scale this picture up to a technicolor theory.

2.1 QCD and Electroweak Symmetry Breaking

Consider a simplified version of the Standard Model containing the full $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance but with no Higgs boson and only a single generation of fermions. In this theory, it would seem that all the fermions are massless and the full $SU(2)_L \times U(1)_Y$ gauge invariance remains manifest. However, this is not quite the case; there would still be electroweak symmetry breaking from QCD confinement and chiral symmetry breaking [7].

To see how this works, note first that the QCD portion of the theory has an approximate $SU(2)_L \times SU(2)_R \times U(1)_V$ global symmetry, of which a $SU(2)_L \times U(1)_Y$ subgroup is gauged. We can identify the gauged $SU(2)_L$ with the corresponding flavour symmetry, while a bit of fiddling shows that hypercharge is generated by

$$Y = t_R^3 + B/2 \ , \tag{40}$$

where B is baryon number and coincides with $U(1)_V$ up to an overall normalization. We can also identify the $U(1)_{em}$ subgroup of the electroweak group to be the portion generated by

$$Q = t_L^3 + Y \ , \tag{41}$$

which is just the standard relation (and consistent with Eq. (26)). Note that this is a subgroup of $SU(2)_V \times U(1)_V$.

As before, $SU(3)_c$ will run strong at low energies and generate a quark condensate as in Eq. (15). This condensate will spontaneously break the axial part of the $SU(2)$ global symmetries. Applying $SU(2)_L$ and $U(1)_Y$ transformations to the quark fields, we also find that the condensate will also break a portion of these as well. Therefore QCD confinement will induce electroweak symmetry breaking! Furthermore, since the $U(1)_{em}$ subgroup is also a subgroup of the unbroken global $SU(2)_V \times U(1)_V$ subgroup, electromagnetism will remain unbroken. Counting degrees of freedom, there are three broken generators and potentially three NGBs. However, we also have $SU(2)_L \times U(1)_Y \to U(1)_{em}$, so three of the NGBs will be eaten to produce massive gauge modes. This means that there won't be any physical NGB modes left over.

To see that the would-be NGBs are eaten, let us try to write an effective theory for them. The leading term that is consistent with the symmetries (in the limit g, $g' \to 0$) and also accomodates the gauged subgroup is

$$\mathscr{L} \supset \frac{f^2}{4} tr\big(D_\mu \Sigma^\dagger D^\mu \Sigma\big) \ , \tag{42}$$

where

$$D_\mu \Sigma = \partial_\mu \Sigma + igt^a W^a_\mu \Sigma - ig' B_\mu \Sigma\, t^3 \ . \tag{43}$$

This form follows from the embedding of the electroweak group in G_{flav}. Note that the B part of the hypercharge component cancels out. Expanding out Eq. (42), the leading terms involving the vector fields are

$$\begin{aligned} \mathscr{L} \quad \supset \quad & \frac{f}{2} g W^+_\mu \partial^\mu \pi^- + \frac{f}{2} g W^-_\mu \partial^\mu \pi^+ + \frac{f}{2}\left(g W^3_\mu + g' B_\mu\right) \partial^\mu \pi^0 \\ & + \frac{f^2}{4}\left[g^2 W^+_\mu W^{\mu -} + (g W^3_\mu - g' B_\mu)^2\right] \ . \end{aligned} \tag{44}$$

The second line gives the usual mass terms for the weak vector bosons, while the bilinear operators in the first line signal that the would-be pions are incorporated into the massive vector bosons as longitudinal components. The resulting W and Z masses are

$$m_W = \frac{g}{2} f \ , \qquad m_Z = \frac{\sqrt{g^2 + g'^2}}{2} f \ , \tag{45}$$

which is just like the usual SM expressions but with $v \to f$.

In contrast to electroweak symmetry breaking by a fundamental Higgs scalar, there is no hierarchy problem for this QCD realization. All the fermions and gauge bosons are effectively massless at high energies by gauge invariance. Going to lower energies, a scale Λ_{QCD} is generated by dimensional transmutation. This scale can be expressed in a suggestive way by rewriting Eq. (13):

$$\Lambda_{QCD} \simeq \Lambda_0 e^{-2\pi / b_{QCD} \alpha_s(\Lambda_0)} \ , \tag{46}$$

where Λ_0 is some fundamental input scale (such as $M_{\rm Pl}$). The large ratio between Λ_{QCD} and the input scale here is a natural consequence of the logarithmic running of $\alpha_s(\mu)$, which gives rise to an exponential of a dimensionless number of order unity.

2.2 Scaling Up QCD to Technicolor

Using QCD for electroweak symmetry breaking does not work in practice for many reasons, including the vector boson masses being much too small (with $f = f_\pi = 94$ MeV). The idea of technicolour (TC) is to add a new non-Abelian gauge group G_{TC} to the SM together with additional *techniquarks* that are charged under both G_{TC} and SM electroweak [7]. If the TC group runs strong above the weak scale, the techniquarks will develop a vacuum condensate that induces the breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ in nearly the same way as QCD in the discussion above. Also as before, this theory does not have an electroweak hierarchy problem.

A minimal TC model consists of a $G_{TC} = SU(N_{TC})$ gauge group with N_f sets of exotic fermions Q_L and Q_R with gauge charges [7]

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} = (N_{TC}, 1, 2, 0) \tag{47}$$

$$Q_R = (U_R, D_L) = (N_{TC}, 1, 1, \pm 1/2) \tag{48}$$

where the brackets refer to $SU(N_{TC}) \times SU(3)_c \times SU(2)_L \times U(1)_Y$. The charges of the exotic *techniquarks* are chosen so that no gauge anomalies arise provided N_{TC} is even.[3] In exactly the same way as for QCD, we can identify the global symmetries of the theory and deduce the set of NGBs that arise from its sponteneous breakdown through fermion condensation.

Let us assume that the $SU(N_{TC})$ gauge coupling runs large to give

$$\langle \overline{Q}_{R_J} Q_{L_I} \rangle = \delta_{IJ}\, \Lambda_{TC}^3 \ , \tag{49}$$

with $I,\ J = 1, 2, \ldots, N_f$ run over all the exotic fermions. If the underlying Lagrangian for these techniquarks contains only gauge-covariantized kinetic terms, it has a global $SU(2N_f)_L \times SU(2N_f)_R \times U(1)_V$ flavour symmetry, of which a $SU(2)_L \times U(1)_Y$ subgroup is gauged (just like in the QCD example above). The techniquark condensate spontaneously breaks the flavour symmetry down to its $SU(2N_f)_V \times U(1)_V$ subgroup. Along the way, the gauged $SU(2)_L \times U(1)_Y$ subgroup is broken down to $U(1)_{em}$. This implies that $(2N_f)^2 - 1$ generators are broken, yielding $(2N_f)^2 - 4$ physical NGBs and three longitudinal vector boson modes for the $W^{\pm}$ and Z^0.

These degrees of freedom can be identified explicitly by writing a low-energy EFT for the $(2N_f)^2 - 1$ would-be NGBs in terms of

$$\Sigma = \exp\left[2i\Pi^a(x) t^a / f_{TC}\right] \ , \tag{50}$$

where now t^a are the generators for the fundamental of $SU(N_{TC})$ and $\Sigma \to L\Sigma R^\dagger$ under the $SU(2N_f)_L \times SU(2N_f)_R$ flavour symmetry and we expect $f_{TC} \sim \Lambda_{TC}$. The leading term in the effective Lagrangian for Σ is

$$\mathscr{L} = \frac{f_{TC}^4}{4} tr\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right) \ , \tag{51}$$

with the covariant derivative now given by

$$D_\mu = \partial_\mu + ig(t^a \otimes \mathbb{I}_{N_f}) W_\mu^a \Sigma - ig' B_\mu \Sigma\, (t^3 \otimes \mathbb{I}_{N_f}) \ , \tag{52}$$

where $(t^a \otimes \mathbb{I}_{N_f})$ is the $(2N_f) \times (2N_f)$ matrix with the 2×2 matrix t^a in each of the $N_f \times N_f$ diagonal sub-blocks. Expanding the Lagrangian term out in terms of component fields, we find something that looks like N_f copies of Eq. (44) but with the pion fields identified with

[3]In the SM, gauge anomalies cancel within each generation between quark and lepton contributions.

composites of the techniquarks rather than the regular quarks. All N_f flavours contribute equally to the NGB modes and the vector boson masses, which are given by

$$m_W^2 = \frac{g^2}{2} N_f f_{TC} \ , \qquad m_Z^2 = \frac{g^2 + g'^2}{2} N_f f_{TC} \ . \tag{53}$$

Since we know the gauge couplings and the vector masses, this lets us fix the scale of techniquark condensation to be on the order of $\Lambda_{TC} \sim f_{TC} = \sqrt{2} m_W / (g\sqrt{N_f}) \sim 100$ GeV.

While this minimal TC theory provides an elegant mechanism for electroweak symmetry breaking, it is also inconsistent with observation several reasons:

1. All the SM fermions are massless, even after EWSB.
2. The theory has massless technipion NGB modes for $N_f > 1$.
3. There is a conserved technibaryon number symmetry, and the lightest technibaryon state is stable and problematic for cosmology [8].
4. It is not clear whether there exists a scalar excitation in the theory that can be idetified with the observed Higgs boson.

Fixing these problems is non-trivial, and seems to force us to extend this simple theory in very complicated ways.

2.3 Extended and Walking Technicolour

Fixing the first three problems with TC listed above can be achieved by connecting the techniquarks to the SM in some. To do this at the renormalizable level, new bosons are needed. Since adding a scalar would likely reintroduce a hierarchy problem, this suggest that it is more promising to connect the two sectors through gauge bosons. This is the approach of extended technicolor (ETC).

In ETC constructions, some part of the SM gauge groups $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ and G_{TC} are embedded within a larger gauge group $G_{ETC} \supset G_{SM} \times G_{TC}$, and techniquarks and SM fermions are joined together within representations of G_{ETC}. At some large scale $\Lambda_{ETC} > \Lambda_{TC}$, a spontaneous breaking of $G_{ETC} \to G_{TC} \times G_{SM}$ is assumed to occur, presumably through a similar fermion condensate mechanism. This yields a set of massive ETC vector bosons with masses on the order of Λ_{ETC}. Integrating them out, one obtains four-fermion operators of the very schematic form

$$-\mathscr{L} \ \supset \ \frac{1}{\Lambda_{ETC}^2}(\overline{Q}Q)(\overline{Q}Q) + \frac{1}{\Lambda_{ETC}^2}(\overline{Q}Q)(\overline{f}f) + \frac{1}{\Lambda_{ETC}^2}(\overline{f}f)(\overline{f}f) \ , \tag{54}$$

where Q is a technifermion and f is a SM fermion. These three operators are each representative of a set of many operators of this general form, and they can be both helpful and dangerous.

The first term in Eq. (54) will generate an explicit mass term for the technipions when the techniquarks condense near Λ_{TC}. This yields technipion masses on the order of

$$m_{\Pi_{TC}} \sim \frac{\Lambda_{TC}^2}{\Lambda_{ETC}} . \tag{55}$$

The second term in Eq. (54) gives explicit SM fermion masses after techniquark condensation on the order of

$$m_f \sim \frac{\Lambda_{TC}^3}{\Lambda_{ETC}^2} \tag{56}$$

These masses are of reasonable size for the lighter generations, but they tend to be too small to account for the third generation. If we try to generate a charm mass from them, $m_c \sim 1$ GeV, this suggests that $\Lambda_{ETC} \lesssim$ TeV once we fix $\Lambda_{TC} \sim 100$ GeV to give the correct vector boson masses. These mixed operators can also allow technibaryons to decay to SM baryons. The third term in Eq. (54) connects SM fermions with each other, and operators of this type can be very dangerous. In particular, they can lead to new sources of flavour violation among the SM fermions. Current bounds on mixing of this type suggest $\Lambda_{ETC} \gtrsim 10^6$ GeV, which is inconsistent with the ETC generation of the charm and strange masses discussed above.

To fix the top mass, different connections of the third generation to the technicolor group have been suggested that allow for additional contributions to the top and bottom masses [9]. The remaining flavour problem for the lighter generations can be addressed by *walking technicolor*, in which strong technicolour renormalization effects between Λ_{ETC} and Λ_{TC} enhance the $(\overline{Q}Q)^2$ and $(\overline{Q}Q)(\overline{q}q)$ operators relative to the $(\overline{q}q)^2$ terms [10]. This enhancement allows Λ_{ETC} to be much larger than one would expect based on the scaling arguments above while still allowing for ETC couplings to generate the first- and second-generation quark masses.

Finally, let us mention that both TC and ETC also have some tension with precision tests of the SM weak sector. This arises from the quantum corrections of the techniquarks (which carry electroweak charges) to the self-energies of the electroweak vector bosons. These corrections are often parametrized in terms of a set of *oblique parameters* S, T, and U. The specific corrections depend on the underlying ETC or TC theory, but they tend to be too large unless there is some degree of seemingly accidental cancellation.

3 Composite and Little Higgs

The fourth problem with TC listed above, the absence of an obvious scalar state that can identified with observed Higgs boson, is not addressed by just extending the TC structure. This suggests that we should try a different approach. A nice way to do this that still relies on new strong dynamics is to generate the Higgs scalar as a composite bound state of more fundamental objects. In these composite scenarios, the resulting Higgs scalar can generate some or all of the fermion masses and electroweak symmetry breaking.

Given the strongly-coupled nature of TC-like dynamics, it is not usually not possible to compute the spectrum or the low-energy couplings analytically. The typical expectation in such theories is that the masses of the bound states should be close to $f_{TC} \sim \Lambda_{TC}$. Suppose we try to identify a Higgs-like scalar H with such a bound state. If $m_H \sim f_{TC}$, there will not be a clear separation of the electroweak symmetry breaking from the Higgs and the contribution from the strong dynamics. Furthermore, $f_{TC} \sim m_W \sim m_h \sim 100$ GeV suggests that it is challenging to satisfy precision electroweak tests.

Most recent attempts to realize the Higgs boson as a composite state try to identify it with an approximate NGB mode [11]. Recall that NGBs, either exact or approximate, are the exceptions to the expectation of $m \sim f_{TC}$ for bound states. A parametrically lighter pseudo-NGB (pNGB) Higgs boson is attractive for two related reasons. First, it gives a natural separation between the weak scale and f_{TC}, and allows us to treat the Higgs field cleanly as a scalar field in an EFT valid below f_{TC}. In this EFT, electroweak symmetry breaking is (mostly) induced by the Higgs VEV in much the same way as in the SM. A second consequence of $f_{TC} > m_W$ is that the corrections to precision electroweak observables are not as large. We will discuss composite Higgs theories in more detail later in the context of theories with extra dimensions.

A related class of theories that relate the Higgs field to an approximate NGB of a spontaneously broken theory are *Little Higgs* models [12, 13]. These are built as effective field theories written in terms of non-linear sigma models with a built-in cutoff $\Lambda \sim 4\pi f$ based up the underlying global symmetry breaking pattern $G \to H$, very much like the effective theories we constructed above for QCD and technicolor. In contrast to QCD or TC, however, LH theories are constructed without reference to the specific dynamics at energies near f. This may be a strong coupling transition or something else. As such, LH theories only push a solution to the full hierarchy problem by an order of magnitude

Little Higgs models also seek to protect the Higgs beyond just the protection from realizing it as an approximate NGB through the mechanism of *collective symmetry breaking* [14, 15]. Consider a theory with a global product group symmetry structure $G = G_1 \times G_2$, of which a subgroup G is gauged such that each G_i factor contains a copy of the SM electroweak group. Upon breaking $G \to H$, the gauged subgroup is broken to the $SU(2)_L \times U(1)_Y$ electroweak group of the Standard Model. Note that by gauging a subgroup of G, the global symmetry is explicitly broken and the would-be NGBs from the spontaneous symmetry breaking at f acquire masses. The Higgs is embedded in this symmetry structure as an exact NGB if one or the other of the gauge couplings g_1 or g_2 of $G_1 \times G_2$ vanishes [14, 15]. Put the other way around, both g_1 and g_2 must be non-zero for the Higgs boson to develop a mass, and thus they collectively break the global symmetry group G. This implies that quadratic corrections to the Higgs mass must vanish at one-loop order, but allows them to return at two loops. The protection at one-loop arises from the cancellation of corrections between SM loops and loops of partner particles required by the underlying global symmetry structure. This is similar to supersymmetry, but now the partners have the same spins as their SM counterparts.

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