P528 Notes #4: Symmetry Breaking

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Symmetries (and gauge invariances) play an extremely important role in physics. The existence of a symmetry constrains what is possible, and this often helps to make calculations easier. And through Noether's theorem, continuous symmetries lead to conservation laws. Symmetries can also be useful when they are approximate, or if they undergo a process called spontaneous symmetry breaking. We examine both possibilities in these notes. We also generalize spontaneous symmetry breaking to gauge invariances to demonstrate the Higgs mechanism.

1 Explicit Symmetry Breaking

Explicit symmetry breaking is just a fancy way of saying that a transformation is not a symmetry at all. However, even when a symmetry is broken explicitly, the machinery we have developed for symmetries can still be useful if there is a sense in which the symmetry breaking is small. In this case, we often say that the would-be symmetry is *approximate*.

To illustrate this, consider the theory of a single complex scalar ϕ with Lagrangian

$$\mathscr{L} \supset |\partial\phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4 - \xi\,\left(\phi^3\phi^* + \phi^{*\,3}\phi\right)\ . \tag{1}$$

We assume that both λ and ξ are real and non-negative. For $\xi \to 0$, this theory has a global symmetry under

$$\phi \to e^{i\alpha}\phi\ , \qquad \phi^* \to e^{-i\alpha}\phi^*\ . \tag{2}$$

The Lagrangian is not invariant under these transformations for non-zero ξ. Correspondingly, the Noether current j^μ for these transformations (defined exactly as before) is not conserved:

$$j^\mu = -i\,\phi^*\overset{\leftrightarrow}{\partial}_\mu\phi\ , \tag{3}$$

has total divergence

$$\partial_\mu j^\mu = -2i\,\xi\left(\phi^3\phi^* - \phi^{*\,3}\phi\right)\ . \tag{4}$$

The degree of non-conservation of the currently is directly proportional to the symmetry breaking parameter ξ.

Now suppose the new dimensionless coupling is very small, $\xi \ll \lambda$. This forces any physical symmetry-breaking processes to be suppressed relative to those that respect the symmetry. In this case, we say that the phase transformation of Eq. (2) is an approximate symmetry. For example, consider the scattering rates for the processes $\phi + \phi \to \phi + \phi$ and

$\phi + \phi \to \phi + \phi^*$ at high energy, $p \gg m$, in the CM frame. The first process respects the approximate symmetry and has an approximate cross section

$$\sigma_1 \sim \frac{\lambda^2}{p^2} \ . \tag{5}$$

The second process does not respect the approximate symmetry, and can therefore only proceed through the coupling ξ, leading to an approximate cross section of

$$\sigma_2 \sim \frac{\xi^2}{p^2} \ . \tag{6}$$

Clearly, we have $\sigma_1 \gg \sigma_2$ for $\lambda \gg \xi$. The idea of an approximate symmetry helps us organize physical processes into those that respect the symmetry and are probable, and those that violate the symmetry and are relatively improbable.

2 Spontaneous Symmetry Breaking

In addition to being broken explicitly, symmetries can also be hidden. This occurs when the underlying action of a theory has a symmetry that is not respected by the vacuum state of the theory. It is often called spontaneous symmetry breaking (SSB), and it has very profound consequences in QFT. We will illustrate the process with a few examples, and then generalize.

2.1 Discrete Symmetries

Consider the Lagrangian

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) \ , \tag{7}$$

with the potential

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4 \ , \tag{8}$$

with μ^2 and λ both positive. The action clearly has a discrete symmetry under $\phi \to -\phi$. However, we also see that the quadratic term does not have the right sign to be a scalar mass term, unless we interpret the mass as $m = i\sqrt{\mu}$. Something is clearly wrong. The way to resolve this can be found by looking at the shape of the potential, which we illustrate in Fig. 1. Evidently the origin of the field space, $\phi = 0$, is not a stable minimum of the potential. For example, solving the classical equation of motion for the scalar starting with $\phi(t = t_0) = 0$, one finds that the amplitude initially grows exponentially (for $\partial_t\phi(t = t_0) \neq 0$). Instead, the stable minima lie at

$$\langle\phi\rangle = \pm\mu/\sqrt{\lambda} \equiv \pm\, v \ . \tag{9}$$

To proceed, we need to add a Rule 0 to the list of rules for dealing with quantum field theories in `notes-00`:

Rules′:

0. Find the global minima of the potential. Choose one of them, and expand in fluctuations around it. The fluctuations should vanish in the vacuum configuration.
1. Start with the quadratic terms in the Lagrangian and extract from them the kinetic and mass terms.
2. For this, redefine the field variables to put the kinetic terms in *canonical* form and then diagonalize the mass matrices.
3. Add the terms higher than quadratic (in terms of the redefined and now-canonical/diagonal fields) and compute perturbatively with Feynman diagrams.

In this example, let us choose to expand around the minimum at $\langle\phi\rangle = +v$:

$$\phi(x) = v + h(x) \ , \tag{10}$$

where $h(x)$ is also a real scalar field. Plugging this form into the original Lagrangian, we see that the kinetic term for $h(x)$ is canonical while the potential becomes

$$\begin{aligned} V &= -\frac{1}{2}\mu^2(v+h)^2 + \frac{\lambda}{4}(v+h)^4 & (11)\\ &= -\frac{1}{4}\lambda v^4 + \frac{1}{2}(2\lambda\, v^2)\, h^2 + \lambda\, v\, h^3 + \frac{\lambda}{4}h^4 \ . & (12)\end{aligned}$$

This potential has a stable minimum at $h = 0$, a sensible mass term for h of $m_h = \sqrt{2\lambda}\, v$, and some h self-interactions.

The potential of Eq. (12) no longer has an obvious reflection symmetry, and is certainly not invariant under $h \to -h$. As a result, we say that the symmetry has been *spontaneously broken*. This is due to the fact that we expanded the theory around a particular vacuum state that does not get mapped back to itself by the symmetry. While this terminology is standard, it is also a bit of a misnomer because the Lagrangian still has a symmetry under

$$h(x) \ \to \ -2v - h(x) \ . \tag{13}$$

In contrast to (most of) the symmetries we studied before, this transformation acts non-linearly on h, since the transformed field is not a linear combination of the original fields due to the constant term. A more accurate description is that the symmetry has been hidden.

At this point you might be wondering why we needed to choose a single vacuum. In ordinary one-particle quantum mechanics, the true ground state for a potential with two equally-deep minima, $|+\rangle$ and $|-\rangle$ say, is a linear combination of the two: $|0\rangle = (|+\rangle + |-\rangle)/\sqrt{2})$. This combined state is invariant under the reflection symmetry, $|+\rangle \leftrightarrow |-\rangle$. Furthermore, the average energy expectation of this state is lower than either of the $|+\rangle$ or $|-\rangle$ states alone (which are not energy eigenstates). If one starts with a system in the $|+\rangle$ state, there is a finite probability to tunnel to the $|-\rangle$ state, and the system can eventually settle down to the true ground state.

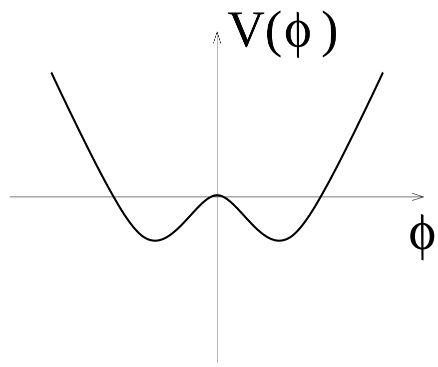

Figure 1: Feynman rules for a non-Abelian gauge theory.

This is not the case for the quantum field theories we study in this course. The essential difference is that these theories are defined in an infinite spacetime volume. Starting with $\phi(x) = +v$, the energy needed to go to $\phi(x) = -v$ is proportional to the volume and is therefore infinite. (The energy cost in one-particle QM is finite.) This implies that it is not possible to tunnel from one vacuum to the other in the field theory in a finite amount of time.[1] As a result, in the QFT we need to choose a single specific vacuum state to expand around. Since the two vacua in this example are physically distinct and separated by an infinite energy cost, expanding about one or the other represents a distinct physical theory. In other words, our QFT is defined both by the Lagrangian of Eq. (7) together with the choice of vacuum state, $|{+v}\rangle$ or $|{-v}\rangle$.

2.2 Continuous Symmetries

Things are even more interesting for continuous symmetries. Consider the $U(1)$-symmetric Lagrangian

$$\mathscr{L} = |\partial\phi|^2 - V(|\phi|) \tag{14}$$

with

$$V(|\phi|) = -\mu^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 \ . \tag{15}$$

This potential obviously has a global $U(1)$ symmetry (as does the kinetic term) and is sometimes called a "wine bottle" or a "Mexican hat". It looks just like that in Fig. 1 after rotating the profile around the vertical axis normal to the $Re(\phi)$-$Im(\phi)$ plane. Recall that the current corresponding to the global $U(1)$ symmetry is

$$j^\mu = -i\,\phi^* \overleftrightarrow{\partial_\mu}\,\phi \ . \tag{16}$$

[1] Even when the volume is finite, we should still work with a single vacuum when the tunnelling time is much longer than all the other relevant time scales in the system.

The global minima of the potential correspond to the base of the wine bottle, and are defined by the condition

$$|\phi|^2 = \mu^2/\lambda \ := \ v^2 \ . \tag{17}$$

Thus, the set of vacuum states is given by

$$\langle\phi\rangle = e^{i\beta}v \quad \leftrightarrow \quad |\beta\rangle. \tag{18}$$

Put another way, we have a circle's worth of distinct vacuum states that we can label by the parameter $\beta \in [0, 2\pi)$. These vacua do not have an energy barrier separating them, but there is an infinite gradient-energy cost to go from one state to another, and we must still pick a specific vacuum to expand around. Any such vacuum breaks the $U(1)$ invariance: $|\beta\rangle \to |\beta + \alpha\rangle$ under $\phi \to e^{i\alpha}\phi$.

Choosing the vacuum state $|\beta\rangle$ for some fixed value of β, we can expand about it by changing our field variables to a polar form:

$$\phi = (v + h(x)/\sqrt{2})e^{i(\beta+\rho(x)/\sqrt{2}v)} \ . \tag{19}$$

This form exchanges the two real degrees of freedom $\{Re(\phi),\, Im(\phi)\}$ for $\{h(x),\, \rho(x)\}$. The advantage of the polar form is that both $h(x)$ and $\rho(x)$ vanish in the vacuum, and therefore represent fluctuations around it (as per Rule 0). In general, you can choose any set of field variables you like as long as they lead to a sensible set of kinetic and mass terms, although a judicious choice can save you a lot of unneeded work. Plugging these new variables into the Lagrangian, we get

$$|\partial\phi|^2 = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(1 + h/\sqrt{2}v)(\partial\rho)^2, \tag{20}$$

as well as

$$V(\phi) = (const) + \frac{1}{2}(2\lambda v^2)h^2 + \frac{\lambda}{\sqrt{2}}vh^3 + \frac{\lambda}{8}h^4. \tag{21}$$

This gives canonical kinetic terms for both h and ρ, some interactions, and masses of $m_h = \sqrt{2\lambda}\,v$ and $m_\rho = 0$. The Lagrangian in this form does not have an obvious rephasing symmetry, so again we say that the theory has undergone spontaneous symmetry breaking (SSB).

The masslessness of $\rho(x)$ here is not an accident. Under $U(1)$ transformations there is still a hidden symmetry under

$$\rho/\sqrt{2}v \to \rho/\sqrt{2}v + \alpha \ , \qquad h \to h \ . \tag{22}$$

In other words, the linear $U(1)$ has become a non-linear shift for ρ. This symmetry forbids non-derivative interactions involving ρ, and thus forbids a mass term for this field. It turns out that this is a generic feature of spontaneously broken continuous symmetries, and the corresponding massless states are called *Nambu-Goldstone Bosons* (NGBs).

It is also instructive to look at the symmetry current in terms of the new field coordinates h and ρ. One finds

$$\begin{aligned} j_\mu &= -i\,\phi^* \overleftrightarrow{\partial}_\mu \phi && (23)\\ &= (v + h/\sqrt{2})^2\, \partial_\mu \rho / \sqrt{2v}\ . && (24) \end{aligned}$$

Compared to the currents we studied before, this current is a bit unusual in that it has a piece that is linear in one of the fields, ρ. Interpreting the current as an operator built from quantum fields, this implies that at leading order

$$\langle 0 | j^\mu(x) | \rho(p) \rangle = \frac{v}{\sqrt{2}}\, p^\mu e^{-ip\cdot x}\ , \tag{25}$$

where $|\rho(p)\rangle$ is a one-particle momentum eigenstate of ρ with 4-momentum p. This matrix element generally vanishes in theories without SSB. Thus, the current is able to create a one-particle NGB state from the vacuum. Taking a partial derivative of this matrix element,

$$\partial_\mu \langle 0 | j^\mu(x) | \rho(p) \rangle = \frac{v}{\sqrt{2}}\, e^{-ip\cdot x} p^2 \ = \ 0\ , \tag{26}$$

where we have used the fact that $p^2 = 0$ for the massless ρ state. Thus, we also find a connection between current conservation and the masslessness of ρ.

2.3 General Nambu-Goldstone Bosons

These results for NGBs are a general feature of continous SSB [1, 2, 3]. To prove this, suppose we have a theory whose Lagrangian is invariant under the continuous (Lie) group G, with a set of scalar fields ϕ_i transforming under some representation of the group. Under an infinitesimal G transformation,

$$\phi_i \to \phi_i + \alpha^a F_i^a(\phi)\ , \tag{27}$$

where α^a are the coordinates of the group transformation.[2] Invariance of the potential under arbitrary G transformations implies that $V(\phi + \alpha^a F^a) = V(\phi)$, which translates into the condition

$$0 = \frac{\partial V}{\partial \phi_j} F_j^a\ , \qquad a = 1, \ldots, d(G). \tag{28}$$

Taking this relation, differentiating it with respect to ϕ_i, and evaluating it at a minimum $\langle\phi\rangle$ of the potential, we get

$$\begin{aligned} 0 &= \left.\frac{\partial V}{\partial \phi_j}\frac{\partial F_j^a}{\partial \phi_i}\right|_{\langle\phi\rangle} + \left.\frac{\partial^2 V}{\partial \phi_j \partial \phi_i} F_j^a\right|_{\langle\phi\rangle} && (29)\\ &= 0 + m_{ij}^2(\langle\phi\rangle)\, F_j^a(\langle\phi\rangle)\ , && (30) \end{aligned}$$

[2] Note that we have changed our notation slightly compared to our previous discussion, $F_i^a = \Delta^a \phi_i$.

where $(...)|_{\langle\phi\rangle}$ implies that one should evaluate the fields at the minimum $\phi = \langle\phi\rangle$. In going to the second line, we have used the fact that the first term vanishes at the minimum of the potential, while the second derivative in the second term corresponds to the scalar mass matrix of the theory when it is expanded around the minimum: $m^2_{ij}(\langle\phi\rangle) = \partial^2 V/\partial\phi_i\partial\phi_j|_{\langle\phi\rangle}$.

The result of Eq. (30) has the form of an eigenvalue equation for the scalar mass matrix for each group generator (labelled by a). It can be solved in two ways. First, the vector $F^a_j(\langle\phi\rangle) = 0$, and no constraint is imposed on the scalar mass matrix. And second, $F^a_j(\langle\phi\rangle)$ is non-zero and corresponds to a zero eigenvalue of the mass matrix. Thus, every non-zero $F^a_j(\langle\phi\rangle)$ implies a massless scalar excitation.

To interpret this result, recall that $F^a(\phi = \langle\phi\rangle)$ are the linear shifts in the field variables at the minimum $\langle\phi\rangle$ under transformations corresponding to a-th generator. Thus, the vacuum state (field minimum) is invariant under small transformations by that generator if and only if the corresponding $F^a(\phi = \langle\phi\rangle)$ vanishes,

$$F^a(\phi = \langle\phi\rangle) = 0 \quad\Longleftrightarrow\quad \textit{the a-th generator leaves the vacuum invariant.} \tag{31}$$

Applying this to Eq. (30), we see that the mass matrix has a zero eigenvalue for every generator that does not leave the vacuum invariant. These zero eigenvalues are precisely the massless NGBs of the theory.

It is easy to count the number of Goldstone modes in a more organized way. The generators that leave the vacuum state invariant form a subgroup H of the bigger symmetry group G. We can choose generators for G such that they can be split according to $\{t^a\} = \{t^A_H, t^B_{G/H}\}$, such that $\{t^A_H\}$ generate the H subgroup that leaves the vacuum invariant, and $\{t^B_{G/H}\}$ make up the rest. The $\{t^A_{G/H}\}$ are said to generate the *coset space* G/H, which may or may not be a subgroup of G. The indices of $\{t^A_H\}$ run over $A = 1, 2, \ldots, d(H)$, and those of $\{t^B_{G/H}\}$ run from $B = d(H)+1, \ldots, d(G)$. Our result above shows that the Goldstone bosons correspond in a one-to-one way with the generators $t^B_{G/H}$ of G/H:

$$\text{NGB} \quad\leftrightarrow\quad \textit{generator of } G/H. \tag{32}$$

There are precisely $[d(G) - d(H)]$ of them. In our previous example, $G = U(1)$ and H was trivial, so $G/H = G$ has one generator, corresponding to the single NGB we found.

Consider now a more complicated example of SSB with the Lagrangian

$$\mathscr{L} = (\partial\phi)^\dagger(\partial\phi) - V(\phi), \tag{33}$$

where

$$V(\phi) = -\mu^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2, \tag{34}$$

and

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \tag{35}$$

is a complex scalar doublet. This theory is invariant under global $SU(2)\times U(1)$ transformations. The minimum of the theory is defined by

$$\phi^\dagger\phi = \mu^2/\lambda \ := \ v^2. \tag{36}$$

The most general vacuum state can be written in the form

$$\langle\phi\rangle = e^{i\beta}e^{i\xi^a t^a}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{37}$$

for some fixed parameters β and ξ^a, with $t^a = \sigma^a/2$ the generators of $SU(2)$.

Let us choose the vacuum state corresponding to $\beta = 0 = \xi^a$. None of the four $SU(2)\times U(1)$ generators leave this vacuum invariant individually, but there is a single linear combination that does:

$$\tilde{t} = \frac{1}{2}\mathbb{I} + t^3. \tag{38}$$

This generates a $U(1)'$ subgroup of $SU(2)\times U(1)$ under which ϕ_+ has charge $\tilde{Q}_+ = 1/2+1/2 = 1$ and ϕ_0 has charge $\tilde{Q}_0 = q/2 - 1/2 = 0$. We identify $G = SU(2)\times U(1)$ and $H = U(1)'$. A set of generators for G/H is

$$\{t^B_{G/H}\} = \left\{\frac{1}{\sqrt{2}}(t^1+it^2),\, \frac{1}{\sqrt{2}}(t^1-it^2),\, (-\frac{1}{2}\mathbb{I}+t^3)\right\}\ . \tag{39}$$

Based on our previous arguments, we expect three NGBs.

There many ways to choose new field variables that will lead to sensible mass and kinetic terms. These choices will result in identical masses but different perturbative couplings. At the end of the day, however, they should all give the same answer for physical observables (although some choices may be much easier to compute with). A convenient choice for this example is

$$\phi(x) = e^{i\rho^B(x)t^B_{G/H}/f}\begin{pmatrix} 0 \\ v + h(x)/\sqrt{2} \end{pmatrix}\ , \tag{40}$$

where $f \sim v$ is a dimensionful constant that we will fix a bit later. Note that there are four real degrees of freedom, the same as initially, and $h = \rho^B = 0$ gives the vacuum state. In this form, it is also clear that the potential depends only on $h(x)$ since all the factors of $\rho^a(x)$ cancel out in $\phi^\dagger\phi$. For the kinetic term, we get

$$(\partial\phi^\dagger)(\partial\phi) \ = \ \frac{1}{2}(\partial h)^2 + (v+h/\sqrt{2})^2\left[\partial\rho^B\partial\rho^C(t^B_{G/H}t^C_{G/H})/f + \ldots\right]_{22}\ . \tag{41}$$

All the terms involving ρ^B involve derivatives, and therefore there is no mass term for these fields. They are evidently the three NGBs of the theory, matching up precisely with the three broken generators. Under infinitesimal G transformations, one also sees that the ρ^B transform by a shift, another tell-tale feature of NGBs.

3 Spontaneously "Broken" Gauge "Symmetries"

Having investigated the spontaneous breakdown of continuous global symmetries, it is natural to do the same for scalar theories with a gauge invariance. The simplest example has a single complex scalar and a $U(1)$ gauge invariance:

$$\mathscr{L} = |(\partial_\mu + igQA_\mu)\phi|^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ , \tag{42}$$

with the same potential as before,

$$V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 \ . \tag{43}$$

The space of vacua is again $\langle\phi\rangle = ve^{i\beta}$. Choosing a fixed value of β, we expand around this vacuum by rewriting the complex scalar as

$$\phi(x) = e^{i(\beta+\rho(x)/\sqrt{2}v)}(v + h/\sqrt{2}) \ . \tag{44}$$

The main difference here compared to the global $U(1)$ theory studied earlier is that we now have the freedom to change the phase of ϕ by an amount that depends on spacetime. In particular, let us make a gauge transformation such that $\beta(x) + \rho(x)/\sqrt{2}v \to 0$ everywhere, or equivalently, $\phi(x) \to (v + h(x)/\sqrt{2})$. Since field configurations related by gauge transformations are physically equivalent, this choice of gauge will not affect our predictions for physical observables.

Expanding the theory in these new variables with this choice of gauge, we find

$$\begin{aligned} |D\phi|^2 &= |(\partial_\mu + igQA_\mu)\phi|^2 \\ &= \frac{1}{2}(\partial h)^2 + \frac{1}{2}\cdot 2\cdot g^2(v + h/\sqrt{2})^2\, A_\mu A^\mu \ . \end{aligned} \tag{45}$$

This yields a nice kinetic term for $h(x)$, but also a mass term for the vector boson with value $m_A = \sqrt{2}gv$, along with some interactions between h and the vectors. However, thinking back to our previous discussion, a massless NGB mode seems to be missing.

To understand what has happened, let us compare the numbers of degrees of freedom (*dofs*) for $\langle\phi\rangle = 0$ and $\langle\phi\rangle \neq 0$. We have:

$$\langle\phi\rangle = 0 : \quad \begin{cases} \phi & \textit{2 real dofs} \\ A_\mu \ (\textit{=massless}) & \textit{2 independent polarizations} \end{cases}$$

$$\langle\phi\rangle \neq 0 : \quad \begin{cases} \phi \to h & \textit{1 real dof} \\ A_\mu \ (\textit{=massive}) & \textit{3 independent polarizations} \end{cases}$$

Aha! $2+2 = 1+3$, and the degrees of freedom match up in both cases. The would-be NGB mode of ϕ has gone to become the longitudinal polarization of the now-massive gauge boson. The highly technical term for this is that the NGB has been *eaten* by the gauge vector to give it mass. This effect is also called the Higgs mechanism, and the remaining physical scalar is called the Higgs boson.[3]

[3] Higgs was one of a number of people to discover this, with others including Anderson, Brout, Englert, Guralnik, Hagen, Nambu, and possibly a few others. Somehow it was Higgs' name that stuck.

4 A Few More Comments

All the theories we have worked with so far have been based on scalar fields. Spontaneous symmetry breaking has corresponded to one or more of them getting non-zero background values in the minimum we chose to expand around. For example, in our initial real scalar theory with a discrete symmetry,

$$\phi(x) = v + h(x) \ , \tag{46}$$

where $h(x)$ corresponded to the physical excitation. As a quantum operator, this implies

$$\langle 0|\phi(x)|0\rangle = v \ , \tag{47}$$

where $|0\rangle$ denotes the vacuum state with zero h particles. For this reason, a non-zero value of a scalar field at a minimum is called a *vacuum expection value* (VEV).

Note that manifest Lorentz invariance forbids a VEV for a single field with non-zero spin. For example, a non-zero VEV for a vector field would pick out a particular direction in spacetime, which we know is not the case (at least to an excellent approximation). However, in theories with strong coupling where perturbation theory breaks down, Lorentz-invariant combinations of fields can obtain VEVs. For example, in low-energy QCD one finds

$$\langle 0|\bar{q}q|0\rangle = \Lambda^3 \neq 0 \ . \tag{48}$$

It turns out that this fermion-bilinear VEV leads to spontaneous symmetry breaking and (approximate) Nambu-Goldstone bosons.

References

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