PHYS 528 Homework #6

Due: Mar.2, 2017

0. Final project topics.

   Choose a topic for the final project. A list of potential topics can be found here:

   http://trshare.triumf.ca/ dmorri/Teaching/PHYS528-2017/fproj.html

   Please check your topic with me before you begin on the project. I will help you get started by suggesting some reading material.

1. Z decays.

   Compute the partial decay widths of the Z^0 into each of the SM fermions. You may ignore the fermion mass whenever it is smaller than one tenth the Z^0 mass: $m_f < m_Z/10$. Note that the branching fraction of a particular decay mode with partial width Γ_i is $BR_i = \Gamma_i/\Gamma_{tot}$. Plug numbers into your formulas and compare to data: *http://pdglive.lbl.gov/*.

2. Invisible Z decays.

   Suppose the SM contains a new "invisible" LH chiral fermion with a coupling strength g_L to the Z^0. (That is, the Lagrangian interaction is $\bar{\psi}_L \gamma^\mu Z_\mu g_L \psi_L$.) Derive the upper limit on the size of the coupling g_L as a function of the fermion mass m_ψ from the requirement that its contribution to the invisible decay width of the Z^0 be less than $\Delta\Gamma_{inv} < 2$ MeV.

3. $e^+e^- \to f\bar{f}$ scattering. ($f \neq e$)

   a) Find the amplitude for this process for a general SM fermion f (that is not the electron e) from the diagram with the photon in the s-channel. Use this to compute the part of summed and squared amplitude from the photon contribution, "$|\mathcal{M}_\gamma|^2$". Here and for the rest of this question, you may neglect fermion masses. Also, show your calculations.

   b) Find the amplitude for this process from the diagram with the Z^0 in the s-channel. Use this to compute "$|\mathcal{M}_Z|^2$" from the Z^0 contribution.
   Hint: there are lots of terms, but many of them vanish!
   Hint: $P_L P_R = 0$, $P_L^2 = P_L$, $P_L\gamma^\mu = \gamma^\mu P_R$, $tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) = -4i\epsilon^{\mu\nu\rho\sigma}$.
   Hint: $\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\rho\sigma} = -2(\delta^\mu{}_\rho\delta^\nu{}_\sigma - \delta^\mu{}_\sigma\delta^\nu{}_\rho)$

   c) Compute the summed and squared interference term "$[\mathcal{M}_\gamma\mathcal{M}_Z^* + (h.c.)]$".

   d) Combine these to find $d\sigma/d\cos\theta$ in the CM frame, where θ is the angle of the outgoing fermion relative to the direction of the incoming electron.

   e) Which contribution dominates for $s = (p_1 + p_2)^2 \ll m_Z^2$? Which dominates for $s \simeq m_Z^2$? What is the asymptotic behaviour for $s \gg m_Z^2$?