PHYS 528 Lecture Notes #9

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1 QCD Overview: Going with the Flow

Quantum Chromodynamics (QCD) is the accepted theory of the strong force. It is an $SU(3)$ gauge theory with matter quark fields transforming under the fundamental **3** irrep of the gauge group. The fundamental Lagrangian is:

$$\mathscr{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \sum_I \bar{q}_I(i\gamma^\mu D_\mu - m_I)q_I, \tag{1}$$

where $I = u,\ d,\ s,\ c,\ b,\ t$, and

$$D_\mu = \partial_\mu + ig_s t^a_3 G^a_\mu, \tag{2}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc}G^b_\mu G^c_\nu\ . \tag{3}$$

The matrices t^a_3 are of course the (eight) generators of the **3** of $SU(3)$. An explicit realization of these matrices can be found on *pg. 44* of Ref. [1]. Gauge charges are called *colour* while the different species of 4-component Dirac fermion quarks are called *flavours*. The masses m_I of the different quark flavours are approximately

$$\begin{array}{lll} m_u \simeq 2.5\text{ MeV}, & m_d \simeq 5.3\text{ MeV}, & m_s \simeq 110\text{ MeV}, \\ m_c \simeq 1.25\text{ GeV} & m_b \simeq 4.5\text{ GeV}, & m_t \simeq 173\text{ GeV}. \end{array} \tag{4}$$

Of course, this structure fits in nicely with the rest of the SM.

While the underlying QCD Lagrangian is very simple, the resulting dynamics are anything but. We never actually observe quarks or gluons as free asymptotic particles. Instead, at low energies (or long distances) we only ever see colour-neutral bound states of quarks and gluons. This stands in stark contrast to QED, where we certainly do see free particles charged under the gauge group – electrons for example. The absence of free colour-charged objects is called *confinement*.

Confinement is still not completely understood at the quantitative level. A large part of the reason for this is the breakdown of perturbation theory in QCD at low energies. Despite these challenges, it is still possible to construct a useful low-energy EFT for the bound states resulting from QCD confinement. Collectively these bound states are called *hadrons*, and the most important examples are *mesons* and *baryons*. The quantum numbers of these states can be matched to the colour-neutral quark operators

$$M \sim \bar{q}^i q'_j \delta^i_j, \qquad B \sim q_i q'_j q''_k \epsilon^{ijk}, \tag{5}$$

where i and j are colour indices.

A very rough idea of where confinement comes from can be obtained by examining the scale dependence of the renormalized QCD coupling $g_s(\mu)$. Recall that, when $\mu \sim p$, the value of this coupling coincides reasonably well with the physical QCD coupling strength in a process occurring at the characteristic momentum scale p. In a generic gauge theory, the running coupling $g(\mu)$ can be obtained by measuring the coupling at one momentum scale and solving the renormalization group (RG) equation to extrapolate it to other momentum scales. At one-loop order, the RG equation is [2]

$$\frac{dg}{dt} \equiv= \beta(t) = -\frac{b}{(4\pi)^3} g^3 \tag{6}$$

where the coefficient b is given by

$$b = \frac{11}{3} C_2(A) - \sum_r \frac{2}{3} T_2(r) - \sum_{r'} \frac{1}{3} T_2(r'), \tag{7}$$

where $t = \ln(\mu/\mu_0)$, the first sum runs over all light 2-component fermion reps in the theory, and the second sum runs over all light complex scalar reps. By "light", we mean all reps with mass $m < \mu$. As μ falls below the mass of a particle in the (effective) theory, we implicitly remove it from the EFT so that it no longer contributes to the RG running. The leading-order matching condition for the running gauge coupling at the mass threshold $\mu = M$ is simply

$$\lim_{\mu \to M_-} g(\mu) = \lim_{\mu \to M_+} g(\mu). \tag{8}$$

That is, the running coupling is continuous across the mass threshold.

In QCD, for $\mu > m_t$ we have

$$b_{QCD} = \frac{11}{3} \times 3 \quad - \quad \frac{2}{3} \times \frac{1}{2} \times 2 \times 6 = 7. \tag{9}$$

In the second term, the 1/2 comes from $T_2(\mathbf{3}) = 1/2$, the 2 comes from the so-called L and R 2-component parts of each quark, and the 6 comes from the six quark flavours. At lower energies, the RG beta-function coefficient becomes

$$b_{QCD} = \begin{cases} 7 & \mu > m_t \\ 23/3 & m_b < \mu < m_t \\ 25/3 & m_c < \mu < m_b \\ \cdots & \end{cases} \tag{10}$$

We can now compute the value of $g_s(\mu)$ at other values of μ after inputing the boundary value of the coupling,

$$\alpha_s(m_Z) \equiv \frac{g_s^2(\mu)}{4\pi} \sim 0.118, \tag{11}$$

into to the solution of Eq. (6).

Note that the sign of the RG coefficient b_{QCD} is such that the strong coupling becomes *weaker* at high energies. This property is called *asymptotic freedom.*[1] The flip side of this is that the QCD coupling grows large at low-energies. We can use this property to derive a dimensionful scale from the dimensionless QCD coupling. The solution to the RG equation (between thresholds) is

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + \frac{b_{QCD}}{2\pi}\ln\left(\frac{\mu}{\mu_0}\right). \tag{12}$$

With this in hand, it makes sense to define the QCD scale Λ_{QCD} as the point where $\alpha_s^{-1}(\mu)$ vanishes. This yields

$$\alpha_s(\mu) = \frac{2\pi}{b_{QCD}\ln(\mu/\Lambda_{QCD})}\ . \tag{13}$$

The appearance of a dimensionful scale from a dimensionless (but scale-dependent) coupling is called *dimensional transmutation.* Numerically, $\Lambda_{QCD} \simeq 200$ MeV, and this value characterizes the onset of strong coupling in QCD. In practice, QCD becomes strongly-coupled a little earlier than this, near $E \sim 1$ GeV, which is roughly the mass scale of the light baryons.

Asymptotic freedom suggests a qualitative picture of confinement. Quarks and gluons are weakly-coupled at high energy, but bind very strongly at low energy as the QCD coupling grows large. Low energies correspond to large distances, and therefore we expect that a quark-antiquark pair will bind together more and more strongly as one attempts to pull them apart. In slightly more detail, the $q\bar{q}$ potential energy at separation r is modelled reasonably well by

$$V(r) \sim -\frac{\alpha_s(r^{-1})}{r} + \Lambda_{QCD}^2 r. \tag{14}$$

The first term is a familiar Coulombic attraction, while the second diverges as $r \to \infty$ and signals confinement. For $r \gtrsim \Lambda_{QCD}$, the energy density between the $q\bar{q}$ pair becomes large enough that it is energetically favourable to nucleate a $q'\bar{q}'$ pair from the vacuum to form a pair of colour-neutral mesons.

The useful degrees of freedom of QCD at energies below Λ_{QCD} are baryons and mesons. We can describe this confined phase of QCD with an EFT based on baryon and meson fields with a natural UV cutoff of roughly Λ_{QCD}. Unfortunately, the UV completion of this theory in terms of quarks and gluons is strongly-coupled at the EFT boundary, and it is not possible to match the two theories in perturbation theory. Instead, one can go surprisingly far in constructing the EFT by using the genuine and approximate symmetries of the underlying theory, which are not expected to be affected (aside from a possible spontaneous breaking) by the strongly-coupled dynamics.

At high energies the relevant degrees of freedom are obviously quarks and gluons. Even so, to probe these objects we are forced to deal with initial states that are colour-neutral composites of these fundamental fields as well as final states that typically bind into colour singlets before we can measure them. Fortunately, the weakness of the QCD coupling at high energies provides a way to factorize the short- and long-distance dynamics.

[1]This property is a very special feature of non-Abelian gauge theories.

2 QCD at Low Energies

At low energies, $E \lesssim 1$ GeV, the QCD degrees of freedom one observes are mainly baryons and mesons. It is therefore much more efficient to describe this system with a field theory that treats these as the dynamical fields instead of using the seemingly more fundamental quarks and gluons. In other words, we want the low-energy EFT of QCD.

This isn't so easy to come by. Quarks and gluons are weakly-coupled at energies well-above Λ_{QCD}, and baryons and mesons are weakly-coupled at energies much below it, but there is strong coupling on both sides where we would like to match them up. Without perturbation theory, one must address the full dynamics of the theory. This is what is done numerically using lattice field theory to simulate the strongly-coupled dynamics of gluons and quarks. A second approach, the one we will discuss here, is to simply write an effective low-energy theory with the appropriate set of degrees of freedom and all possible interactions consistent with the underlying symmetries [3, 4, 5].

Relative to Λ_{QCD}, the u and d quarks are both very light, the s quark is somewhat light, and the other quarks are relatively heavy. Thus, to study the lightest QCD degrees of freedom we should be able to integrate out the c, b, and t quarks and work only with the u, d, and s quarks. To simplify the discussion here, let's also ignore the s quark for now and treat the u and d quark masses as small corrections that we will handle perturbatively.

With only the u and d quarks and neglecting their very small masses (relative to $\Lambda_{QCD} \sim 200$ MeV), the QCD Lagrangian of Eq. (1) has a global $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$ flavour symmetry under which the fields transform as

$$q_{L\,I} \to e^{i\alpha_V} e^{i\alpha_A} L_{IJ} q_{L\,J}, \qquad q_{R\,I} \to e^{i\alpha_V} e^{-i\alpha_A} R_{IJ} q_{R\,J}, \tag{15}$$

where $q = (u,\, d)^t$, $I,\, J = u,\, d$, and L and R are $SU(2)$ transformations for the fundamental rep in flavour space. Of the factors making up this global symmetry group, only the $U(1)_A$ part is anomalous with respect to QCD ($SU(3)_c$), meaning that the remaining factors are all good symmetries at the quantum level. Therefore we should try to build a low-energy effective theory that is symmetric under $G_{flav} = SU(2)_L \times SU(2)_R \times U(1)_V$.

Before attempting to write down such a theory, let us mention one additional and essential fact: strong coupling in QCD generates an expectation value for the gauge-invariant $\bar{q}q$ quark operator:

$$\langle \bar{q}_{R\,J} q_{L\,I} \rangle = \Lambda^3_{QCD} \delta_{IJ}, \tag{16}$$

where I and J run over u, d, and s. This quark condensate expectation value *does not* respect the full (non-anomalous) global symmetry group. Applying a general G_{flav} transformation to this operator, the expectation value is not invariant and changes into

$$\Lambda^3_{QCD} \delta_{IJ} \to \Lambda^3_{QCD} (LR^\dagger)_{IJ}. \tag{17}$$

Thus, the $\bar{q}q$ expectation value spontaneously breaks G_{flav} to a smaller subgroup. It is not hard to see that this subgroup is $H_{flav} = SU(2)_V \times U(1)_V$, where $SU(2)_V$ is the subgroup of $SU(2)_L \times SU(2)_R$ transformations with $L = R$. The global G_{flav} symmetry is sometimes called a *chiral* symmetry, and its breaking is referred to as *chiral symmetry breaking.*

This spontaneous breakdown of $G_{flav} \to H_{flav}$ has three corresponding broken generators. Therefore we expect three massless Nambu-Goldstone bosons (NGBs). Since the other QCD degrees of freedom are generically expected to pick up masses on the order of Λ_{QCD}, it makes sense to build a low-energy EFT with only these NGBs as the light degrees of freedom. Once we do, we will try to identify these light NGB fields with observed particles. The unbroken $SU(2)_V$ symmetry is called *isospin*, while the unbroken $U(1)_V$ corresponds to baryon number (up to an overall normalization of the generators). Since chiral symmetry breaking plays an essential role in constructing this EFT, it is usually called *chiral perturbation theory.*

There isn't a unique way to build the EFT for the NGBs, but we should at least make sure the EFT respects the full underlying G_{flav} global symmetry, has three explicit degrees of freedom, and that the corresponding field excitations vanish in the vacuum configuration of the theory. A convenient way to accomplish these tasks is to use field variables that look like spacetime-dependent G_{flav} transformations acting on the vaccuum. Here, this corresponds to building the theory out of the 2×2 matrix of fields

$$\Sigma(x) = exp[2i\Pi^a(x)t^a/f], \tag{18}$$

where $t^a = \sigma^a/2$, the $\Pi^a(x)$ are the dynamical fields, and f is an as-yet-unspecified parameter with dimension of mass. Under G_{flav} transformations, this field matrix is assumed to transform as

$$\Sigma(x) \to L\,\Sigma(x)\,R^\dagger. \tag{19}$$

Thus, $\Sigma(x)$ transforms in the same way as the quark condensate, Eqs. (16,17).

It is instructive to look at how the Π fields transform under the action of G_{flav} and H_{flav}. For this it is useful to write (without loss of generality)

$$L = e^{ic_A^a t^a} e^{ic_V^b t^b}, \qquad R = e^{-ic_A^a t^a} e^{ic_V^b t^b}. \tag{20}$$

In this form we see that $SU(2)_V$ coincides with $c_A^a = 0$. Acting with an infinitesimal $SU(2)_V$ transformation on Σ ($L = R \equiv V$) we find that

$$\Sigma(x) \to V\Sigma V^\dagger = exp[2i\,V\Pi^a t^a V^\dagger/f], \tag{21}$$

and that

$$\Pi^a \to \Pi'^a = \left(\delta^{ac} - f^{abc}c_v^b\right)\Pi^c + \mathcal{O}(c_V^2). \tag{22}$$

Thus Π transforms linearly and in the adjoint representation of $SU(2)_V$. Under transformations by the broken generators (*i.e.* $c_V = 0$), we find that

$$\Pi \to \Pi' = \Pi + f c_A^a t^a + \mathcal{O}(c_A^2). \tag{23}$$

This is a non-linear transformation on Π, and it takes precisely the shift form we expect for a Goldstone boson field. These nice transformation properties are the reason why the seemingly funny choice of field variables made in Eq. (19) is so useful.

We can now write down a Lagrangian making use of the nice field variables Σ. Even though part of G_{flav} is spontaneously broken, the low-energy effective Lagrangian should still be symmetric under the full group. Looking at reasonable real combinations of Σ, we see that $\Sigma^\dagger\Sigma = \mathbb{I} = \Sigma\Sigma^\dagger$ is trivial. To get something non-trivial we need derivatives. The lowest-order term is

$$\partial_\mu\Sigma^\dagger\partial^\mu\Sigma \to R(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma)R^\dagger \tag{24}$$

implying that $tr(\partial\Sigma^\dagger{\cdot}\partial\Sigma)$ is real and invariant under the symmetries. Thus, the lowest-order term one can write within this theory is

$$\begin{aligned}\mathscr{L}_{p^2} &= \frac{f^2}{4}\,tr(\partial\Sigma^\dagger{\cdot}\partial\Sigma) \\ &= \frac{1}{2}(\partial\Pi^a)^2 + \frac{1}{3f^2}tr[(\partial\Pi\cdot\Pi)^2] + \ldots\end{aligned} \tag{25}$$

The first term is a canonical kinetic term for the Π fields while the second is an interaction term. Relative to the first term, the leading interaction is suppressed by a factor of p^2/f^2. Higher-order terms in the expansion of this operator are suppressed by additional powers of p^2/f^2. Thus, this theory is only useful as an EFT valid for $p^2 \ll f^2$.

The next set of terms we can write down come in with suppressions of at least p^4/f^4. They are

$$\begin{aligned}\mathscr{L}_{p^4} = \;& L_1\left[tr(\partial\Sigma^\dagger{\cdot}\partial\Sigma)\right]^2 \\ &+ L_2\,tr(\partial_\mu\Sigma^\dagger\partial_\nu\Sigma)\,tr(\partial^\mu\Sigma\partial^\nu\Sigma) \\ &+ L_3\,tr(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma\,\partial_\nu\Sigma^\dagger\partial^\nu\Sigma),\end{aligned} \tag{26}$$

where L_1, L_2, and L_3 are unknown dimensionless coupling constants. Note that all these terms involve only derivatives of Σ, which is required by the non-linear transformation properties of this field under transformations induced by the spontaneusly broken generators. There is an infinite set of even higher-order terms that can be added. In practice, however, as long as $p^2 \ll f^2$ and we only demand a finite level of accuracy in our theoretical predictions, only a finite set of operators need be considered.

With a sensible EFT in hand, the next step is to connect the dynamical fields it contains to physical particles and use this connection to fix the numerical value of f (and the other couplings). We can identify the electromagnetic charges of the Π^a fields by noting that a subgroup of $SU(2)_V \times U(1)_V$ coincides with (spacetime independent) QED gauge transformations. The corresponding generator is[2]

$$Q \equiv t^3_V + \frac{1}{6}\mathbb{I}. \tag{27}$$

Applying such a transformation to Π, we find that its components have electric charges $Q = 0, \pm 1$, with

$$\pi^0 = \Pi^3, \qquad \pi^\pm = \sqrt{2}(\Pi^1 \mp i\Pi^2). \tag{28}$$

[2]We are treating QED as a small perturbation on QCD here, and it should be clear that this exact gauge symmetry explicitly breaks the global flavour symmetries we are discussing by a small amount.

It is natural to identify these states with the lightest strongly-interacting colour-singlet paticles: the neutral and charged pions. Furthermore, the pions are known to be pseudoscalars, which is also true for NGBs. At this point the pions in our EFT are exactly massless whereas the real pions have masses of about 135 MeV. We will see shortly how to account for this apparent discrepancy.

We would also like to fix the dimensionful parameter f in our theory. For this, we can match the conserved current operators in both the underlying QCD theory of quarks and gluons with the current operators in the pionic EFT. In the quark theory we have

$$\begin{aligned} j_V^\mu &= \bar{q}\gamma^\mu q, & j_A^\mu &= \bar{q}\gamma^\mu\gamma^5 q \\ j_L^{a\,\mu} &= \bar{q}\gamma^\mu P_L t^a q, & j_R^{a\,\mu} &= \bar{q}\gamma^\mu P_R t^a q. \end{aligned} \tag{29}$$

In the EFT, we find using Noether's theorem

$$\begin{aligned} j_V^\mu &= 0 \\ j_L^{a\,\mu} &= -i\frac{f^2}{2}tr(\Sigma^\dagger t^a \partial^\mu \Sigma) = f\,tr(t^a\partial^\mu\Pi) + \mathcal{O}(\Pi^2), \\ j_R^{a\,\mu} &= -i\frac{f^2}{2}tr(\Sigma\, t^a \partial^\mu \Sigma^\dagger) = -f\,tr(t^a\partial^\mu\,\Pi) + \mathcal{O}(\Pi^2). \end{aligned} \tag{30}$$

Consider now the decay of a negatively charged pion. This proceeds through a W^-, and its amplitude is proportional to the matrix element

$$\langle \mu^-\bar{\nu}_\mu\,|\mathcal{H}_{int}|\,\pi^-(p)\rangle \tag{31}$$

with the interaction operator given by

$$\mathcal{H}_{int} = \frac{g^2}{2m_W^2}(\bar{u}\gamma^\mu P_L d)\,(\bar{\mu}\gamma_\mu P_L \nu_\mu). \tag{32}$$

Contracting fields with external states, the matrix element factorizes into a simple leptonic piece, and a complicated hadronic piece given by

$$\langle 0\,|\bar{u}\gamma^\mu P_L d\,|\pi^+(p)\rangle \equiv i\frac{1}{\sqrt{2}}\,f_\pi p^\mu, \tag{33}$$

where the right-hand side is fixed by Lorentz invariance. Now, we can write this quark operator in terms of a current

$$\bar{u}\gamma^\mu P_L d = \left(j_L^{1\,\mu} + j_L^{2\,\mu}\right) = \frac{1}{\sqrt{2}}\,f\,\partial^\mu\pi^- + \mathcal{O}(\pi^2). \tag{34}$$

Plunking this into the pion matrix element, we see that to leading order

$$f = f_\pi \simeq 93 \text{ MeV}, \tag{35}$$

where the latter numerical value is extracted from the measured the rate of pion decays. Measurements of pion scattering yield values of L_1, L_2, and L_3.

The last piece of the puzzle is explaining the pion masses. Recall that in setting up our EFT for pions we purposely ignored the small u and d quark masses. These can be put back in and treated as small perturbations since $m_{u,d} \ll f$. Even though they are small, these masses play an essential role because they *explicitly* break $SU(2)_L \times SU(2)_R$ down to $SU(2)_V$ for $m_u = m_d$, and down to nothing at all for $m_u \neq m_d$ (which seems to be the case). As a result of this explicit breaking, our $SU(2)_L \times SU(2)_R$ global symmetry is only approximate, with symmetry breaking effects on the order of $m_{u,d}/f$. The would-be NGB pions are now only pseudo-NGBs that have small but non-zero masses proportional to m_u and m_d.

We will write the quark mass matrix as $M = diag(m_u, m_d)$, so that

$$-\mathscr{L} \supset \bar{q}_R M q_L + \bar{q}_L M^\dagger q_R. \tag{36}$$

If this fixed matrix did transform along with the quark fields under G_{flav} according to

$$M \to R M L^\dagger, \tag{37}$$

we would regain the full G_{flav} invariance. Of course it doesn't, but if we pretend it does and impose G_{flav} symmetry with this imagined transformation law, we can keep track of the symmetry breaking effects in an organized way. The leading EFT term that can be written with this in mind is

$$-\mathscr{L} \supset \frac{1}{2} f^2\, tr(\tilde{\Lambda} M \Sigma) + h.c. \tag{38}$$

where we expect $\tilde{\Lambda} \sim \Lambda_{QCD}$. Expanding this out, we find a pion mass term of

$$m_\pi^2 = \tilde{\Lambda}(m_u + m_d). \tag{39}$$

As expected, the pion masses go to zero as the underlying quark masses vanish.

So far we have neglected the strange quark, but it turns out to be a pretty good approximation to include it as well, treating its mass as another small perturbation. The resulting theory now has an approximate $SU(3)_L \times SU(3)_R \times U(1)_V$ global symmetry that is spontaneously broken by the QCD vacuum down to $SU(3)_V \times U(1)_V$. This produces an octet of eight (pseudo-) NGBs that can be identified with the pions and kaons. More precisely, the components of Σ now correspond to

$$\Pi^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}. \tag{40}$$

As before, we can derive the approximate masses of these states by adding the 3×3 mass matrix $M = diag(m_u, m_d, m_s)$ to the theory as a small perturbation. These masses agree pretty well with the observed values. A different set of technology is needed to describe mesons involving c and b quarks. The top quark, being very heavy, decays too quickly to form meson bound states.

Low-energy QCD also involves baryons. These cannot be identified as NGB modes, and are somewhat harder to describe. The presence of an approximate global $SU(3)_V \times U(1)_V$

still turns out to be very useful in organizing the various baryon states, and the lowest-lying modes naturally fill out singlet and octet representations. These symmetries also constrain the form of interactions between baryons and mesons, and symmetry breaking effects can again by added perturbatively.

Before finishing up, let us mention a couple of additional points. First, we have not looked into the effects of electromagnetism on the chiral perturbation theory EFT we have discussed. Relative to the dominant QCD dynamics, we can treat QED effects as small perturbations to the leading behaviour we have discussed here. In a few cases, however, QED effects can be very important. In particular, the G_{flav} global symmetry we have discussed is broken explicitly by QED (*e.g.* the u and d quarks transforming as $SU(2)_{L,R}$ doublets have different QED charges), and it also has an anomaly with respect to QED. The explicit breaking leads to electromagnetic contributions to the pion masses that split the values of the charged and neutral states. The anomaly in G_{flav} relative to QED leads to a coupling between the π^0 and two photons. It turns out that this anomaly-induced coupling leads to the dominant decay channel of the neutral pion: $\pi^0 \to \gamma\gamma$ has a branching fraction of nearly 99%.

Second, going beyond chiral perturbation theory, there are also many heavier QCD excitations. Even though it is very difficult to predict their masses, we do expect them to respect the approximate G_{flav} global symmetry and to appear in complete representations of the unbroken H_{flav} subgroup.

References

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