PHYS 528 Lecture Notes #7

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February 23, 2011

1 Electroweak Tests of the SM

The electroweak sector of the Standard Model (SM) has been tested extensively in both high-energy particle collisions and lower-energy precision probes. These tests have given us confidence that a hidden $SU(2)_L \times U(1)_Y$ gauge invariance underlies both the electromagnetic and weak forces. In the present section we will discuss how to connect the SM at the theoretical level to experimental observables. To begin, we will restrict ourselves entirely to tree-level expressions, but we shall discuss the importance of loop corrections at the end.

The electroweak sector of the SM Lagrangian is completely characterized by three indepedent parameters which can be chosen to be $\{g,\, s_W,\, v\}$. Once we fix the values of these parameters, by making three independent experimental measurements, we can predict all the other electroweak observables in the SM [1, 2]. It is convenient to choose these "input" observables to be α_{em}, G_F, and m_Z which are among the best-measured quantities.

The electromagnetic coupling α_{em} is determined at low energy from the anomalous magnetic moment of the electron, and is then extrapolated up to the value relevant for physics at energies close to m_Z. The current status is [3, 4]

$$\begin{aligned} \alpha_{em}(m_Z) &\equiv \frac{e^2}{4\pi} = \frac{g^2 s_W^2}{4\pi} \\ &= (127.92 \pm 0.02)^{-1} \end{aligned} \tag{1}$$

Muon decays are used to extract the Fermi constant G_F, which is given by

$$\begin{aligned} G_F &\equiv \frac{1}{2\sqrt{2}\,v^2} \\ &= (1.166364 \pm 0.000005) \times 10^{-5}\ \text{GeV}^{-2}. \end{aligned} \tag{2}$$

For the mass of the Z^0, it is obtained from the energy dependence of the cross-section for $e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\begin{aligned} m_Z^2 &= \frac{g}{\sqrt{2}\,c_W}\,v \\ &= (91.188 \pm 0.002)\ \text{GeV}. \end{aligned} \tag{3}$$

It is straightforward to solve for g, s_W, and v from these expressions.

Having fixed the input values of the Lagrangian, we can now go on to compute any other electroweak observable we would like. The most useful of these are related to processes of the form $e^+e^- \to f\bar{f}$ at centre-of-mass (CM) energies near the Z^0 mass, $s = (p_{e^-} + p_{e^+})^2 \simeq m_Z^2$. In this regime, the dominant contribution to the cross section comes from the diagram with a Z^0 in the s-channel.[1] The dominance of the Z^0 diagram comes about because of the form of the Z^0 propagator denominator appearing in the amplitude:

$$\mathcal{M} \propto \frac{1}{p^2 - m_Z^2}. \tag{4}$$

This evidently has a pole and blows up for $s = p^2 = m_Z^2$.

The divergence here is not physical, however. Adding quantum corrections to the propagator, primarily in the form of fermion loops, the propagator denominator acquires an imaginary piece that is approximated well by [5, 6]

$$\frac{1}{p^2 - m_Z^2} \to \frac{1}{p^2 - m_Z^2 + i\sqrt{p^2}\,\Gamma_Z}, \tag{5}$$

where Γ_Z is the *total* decay width of the Z^0 into all possible final states. That the new contribution is imaginary means that it represents an absorptive effect in the propagation of the Z^0, anc corresponds to the loss of propabability amplitude for the Z^0 to keep propagating along, due to its finite lifetime ($\tau_Z = 1/\Gamma_Z$) to decay into other particles. Even though the present discussion deals with tree-level quantities, we will include this finite-width correction to avoid the unphysical divergence.

The master equation for computing electroweak observables is the differential cross-section for $e^+e^- \to f\bar{f}$ in the CM frame, and is given by (neglecting light fermion masses)

$$\begin{aligned}\frac{d\sigma}{d\cos\theta} = & \frac{1}{64\pi}\frac{s}{(s-m_Z^2)^2 + s\Gamma_Z^2} \\ & \big[(|a_{LL}|^2 + |a_{RR}|^2 + |a_{LR}|^2 + |a_{RL}|^2)(1+\cos^2\theta) \\ & +(|a_{LL}|^2 + |a_{RR}|^2 - |a_{LR}|^2 - |a_{RL}|^2)(2\cos\theta)\big]\,,\end{aligned} \tag{6}$$

where $\cos\theta$ is the scattering angle of the f fermion relative to the electron beam and the coefficients are

$$a_{AB} = g_A^e g_B^f, \tag{7}$$

with A, $B = L$, R. Recall that $g_A^f = (g/c_W)(t^3 - Qs_W^2)$.

An obvious first thing to look at is the energy dependence of the cross-section for $\sqrt{s} \sim m_Z$. The LEP-I (CERN) and SLC (SLAC) experiments did just this and found a clear mass peak, as shown in Fig. 1. Based on the location and shape of the peak, it is possible to extract m_Z and Γ_Z (under the SM hypothesis). We used the extracted value of m_Z as an input, but $\Gamma_Z = (2.495 \pm 0.002)$ GeV is an independent observable whose value we can compute.

After figuring out the location of the Z pole, the LEP-I and SLC (and SLD) experiments ran primarily on the pole with $\sqrt{s} = m_Z$ or at least as close as they could get. In this

[1] For $f \neq e$ the other tree-level diagram has an s-channel photon. There are also additional t-channel photon and Z diagrams for $f = e$.

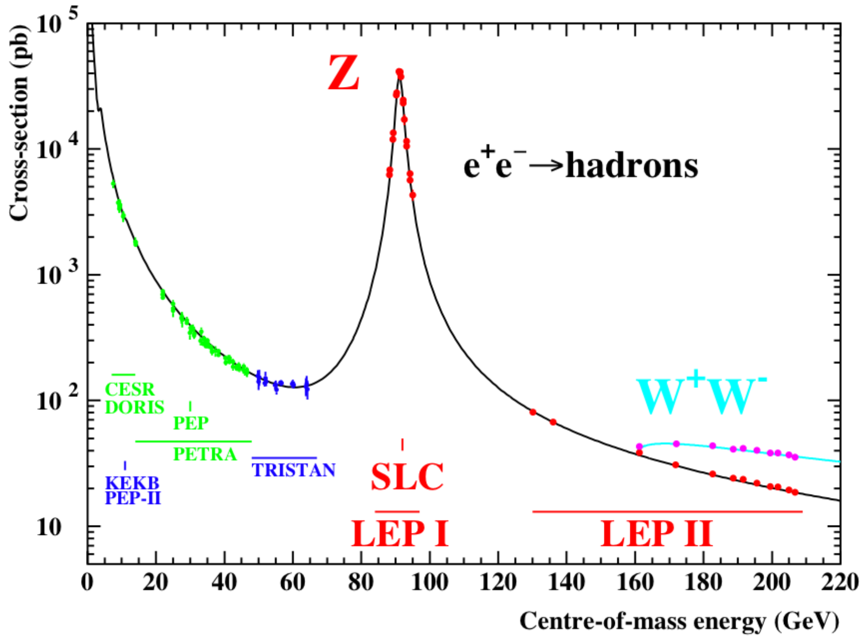

Figure 1: Cross section for $e^+e^- \to hadrons$ at various energies.

situation we can think of $e^+e^- \to f\bar{f}$ as $e^+e^- \to Z^0$ followed by $Z^0 \to f\bar{f}$. The relative cross-sections for different fermion final states are then proportional to the partial decay widths of the Z^0 into these final states. This allows us to extract the branching fractions of the Z^0,

$$BR_f \equiv \Gamma(Z \to f\bar{f})/\Gamma_Z \equiv \Gamma_f/\Gamma_Z, \tag{8}$$

which we can now compare to the values predicted by the SM. Sometimes you will see various R_f quantities defined according to

$$R_\ell = \Gamma_{had}/\Gamma_\ell \quad (\ell = e, \mu, \tau), \qquad R_b = \Gamma_b/\Gamma_{had}, \qquad R_c \equiv \Gamma_c/\Gamma_{had}, \tag{9}$$

where Γ_{had} the decay width in all the kinematically accessible quarks.

Unlike all the other Z^0 decay channels, the neutrino final states are not seen directly. Instead one can deduce the total *invisible* partial width of the Z^0 by using

$$\Gamma_{inv} = \Gamma_Z - \Gamma_e - \Gamma_\mu - \Gamma_\tau - \Gamma_{had} \tag{10}$$

Comparing Γ_{inv} to the SM prediction for neutrinos, the data only matches what is seen if there are three (active) neutrino species. The current experimental uncertainty in Γ_{inv} implies that any additional (non-SM neutrino) invisible Z^0 decay channels must have a total width less than $\Delta\Gamma_{inv} \lesssim 2$ MeV.

Besides just branching fractions and overall cross-sections, there is additional information to be had in angular distributions and spins. The left-right asymmetry A_f is defined to be

$$A_f \equiv \left[\Gamma(Z \to f_L\bar{f}_R) - \Gamma(Z \to f_R\bar{f}_L)\right]/\Gamma(Z \to f\bar{f}). \tag{11}$$

Here, $\bar{f}_R$ is the right-handed anti-fermion conjugate of f_L, and could more properly be written as $\overline{(f_L)}$. [2] It is not hard to show that

$$A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2} \tag{12}$$

It is possible to measure these left-right asymmetries by using polarized electron beams at the Z pole which was done at the SLAC SLD experiments. Note that sometimes one also sees $A_f = A_{LR}^f$.

The forward-backward asymmetry in $e^+e^- \to f\bar{f}$ is defined by

$$A_{FB}^f \equiv \frac{\left(\int_0^1 - \int_{-1}^0\right) d(\cos\theta)\ \frac{d\sigma(e^+e^- \to f\bar{f})}{d(\cos\theta)}}{\left(\int_0^1 + \int_{-1}^0\right) d(\cos\theta)\ \frac{d\sigma(e^+e^- \to f\bar{f})}{d(\cos\theta)}}. \tag{13}$$

This quantity is a function of the CM energy $\sqrt{s}$, but it is usually quoted for $\sqrt{s} = m_Z$. On the Z pole one can show that

$$A_{FB}^f = \frac{3}{4} A_e A_f. \tag{14}$$

In addition to these primarily Z-pole observables, there are also some very good tests of the electroweak structure of the SM at both lower and higher energies. At lower energies neutrino cross-sections, measurements of atomic parity violation, and the determination of the τ lifetime are especially important. Higher energy colliders, and the Tevatron in particular, have measured m_W very precisely [7]:

$$m_W = (80.40 \pm 0.02)\ \text{GeV} \tag{15}$$

These colliders have also measured rates of W and Z production, which can again be compared with the predicted values within the SM.

2 The Current Status of the EW Sector of the SM

Putting the axe to the grindstone, that is comparing the predictions of the SM to experimental observations, the theory does extremely well in nearly every regard. As such, the SM really does appear to be the correct theory of Nature at energies below about 200 GeV. The only missing component of the SM is the Higgs boson, but as will see below, we have very good indirect evidence for its existence as well.

The current set of experimental tests of the electroweak sector of the SM are so good that small perturbative loop corrections must be included in the theoretical predictions in order to achieve the same level of precision. The most important contributors to these loops

[2] Recall that the conjugate of a 2-component LH fermion is a RH 2-component fermion.

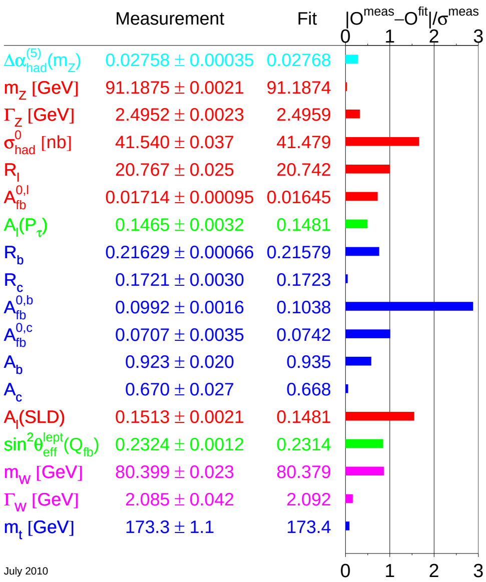

Figure 2: Observed values of various electroweak observables compared to the best-fit predictions of the SM.

are the top quark and the Higgs boson. As such, the theoretical predictions of the SM are very sensitive to the values of the top quark and Higgs boson masses. The measured value of the top mass is [8]

$$m_t = (173.1 \pm 1.1)\ \text{GeV}. \tag{16}$$

We unfortunately do no know the Higgs boson mass, but a value of $m_h = 114$ GeV is typically chosen as a fiducial value.

With these values in place, together with a good deal of careful theoretical work in calculating the observables, the net global fit works pretty well. The way specific measurements match up with the global is shown in Fig. 2 [4]. The largest statistical *pull* comes from A_{FB}^b, but it is not unreasonably large given the number of independent measurements.

In making theoretical predictions for the SM we included a Higgs boson and we fixed its mass to $m_h = 114$ GeV. Since we haven't yet discovered this state, we should also look at what happens to the fit to electroweak observables as we vary the Higgs mass. The result is shown in Fig. 3 [4]. This so-called blue-band plot shows that the fit gets increasingly worse for larger Higgs masses. The choice of 114 GeV happens to be the lowest value the Higgs mass can take while remaining consistent with existing searches for this particle. The fact that we need to include a Higgs in the EW fit and to fix its mass to a relatively low value suggests quite strongly that the Higgs boson really does exist and that we will discover it before too long. Hooray!

References

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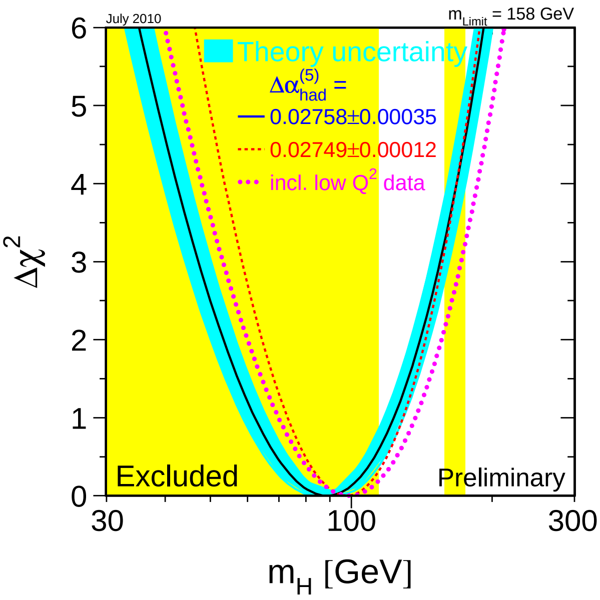

Figure 3: Dependence of the goodness of the electroweak fit on the Higgs boson mass.

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[3] K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. G **G37**, 075021 (2010).

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See also: http://lepewwg.web.cern.ch/LEPEWWG/

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