Neutrino Masses and Mixings, and Current Experimental Situation

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1 Introduction

Neutrino oscillation is the first known case where there is a good evidence that the standard model is an incomplete theory. The standard model predict that the lepton number L is conserved and as a consequence the neutrinos are massless. Also each ν_l only participate in the charged-current weak interaction together with its corresponding charged lepton.

Recent data from neutrino experiments involving neutrinos of relatively low energy, propagating over long distances, show that the lepton flavour is not conserved and neutrinos of one flavour oscillate into neutrinos of other flavours. However we could extend standard model to permit the neutrino masses that can describe the observed neutrino oscillation.

2 Neutrino masses

Standard model is the most general theory which is consistent with general principles plus two assumption: renormalizability and the given particle content. Since the experiments show that this theory is not complete, one of this two assumptions must be breaking down. So there are two way to extend the standard model: either introduce additional light degrees of freedom or allow the non-renormalizable interactions in the theory.

2.1 Sterile neutrinos

We start by considering that the set of fermionic fields chosen for the standard model is incomplete. Since LEP (Large Electron-Positron Collider) excluded new particles coupled to the Z boson and lighter than $M_z/2$, we can only add light right-handed neutrinos ν_R. These new fields must be singlet under all $SU_c(3) \times SU_L(2) \times U_Y(1)$ in order to allow the gauge invariant Yukawa interaction with the lepton doublet L, in other words they must be sterile.

$$L_N = -\frac{1}{2}\bar{\nu_r} i\partial\!\!\!/ \nu_R - \frac{1}{2}M\bar{\nu_R}\nu_R - (\lambda_N \bar{L}\tilde{\phi}\nu_R + h.c.)$$

Gauge invariance allows a Majorana mass term $\frac{1}{2}M\bar{\nu_R}\nu_R$ that breaks lepton number (It breaks the total lepton number by two units). Neutrinos can have Dirac masses like all other fermions if we impose the conservation of lepton number by hand (in other words if we set $M = 0$). In this case the neutrino Yukawa coupling gives the Dirac neutrino mass $m_\nu = \lambda_N v \approx 0.1eV$ for $\lambda \sim 10^{-12}$.

2.2 Higher dimension interactions

Next suppose that we do not enlarge the field content of SM. We can still explain the neutrino oscillations by allowing our theory to have non-renormalizable interactions. These interactions could be the consequence of new physics at high energy and the standard model is just the low energy effective theory of this new physics. We need to find the lowest-dimension non-renormalizable interactions. There is only one kind of dimension-5 operator which is allowed by the SM particle content and gauge symmetries: $\tilde{\phi}_\alpha \bar{L}^\alpha L^\beta \tilde{\phi}_\beta$.

Inserting the Higg vev v, this operator gives a Majorana neutrino mass term ,$\frac{1}{2}m_\nu \bar{\nu_L}\nu_L$, for left-handed neutrinos with $m_\nu = \frac{v^2}{\Lambda} \approx 0.1eV$ for $\Lambda \sim 10^{14}GeV$. Since this scale is so large we can neglect the still higher dimension interactions. It also shows that if there are only three neutrinos, the fact that their masses are so small is just a natural consequence of the scale of the new lepton-number-violating physics being very large.

2.2.1 The see-saw mechanism

We can ask what is the possible origin of this 5-dimension operator. However we cannot discriminate different sources since whatever is the source of $\tilde{\phi}_\alpha \bar{L}^\alpha L^\beta \tilde{\phi}_\beta$ operator, this operator is all what we can see at low energies.

Tree level exchange of three different types of new particle can generate neutrino masses: right-handed neutrinos and fermions or scalar $SU(2)_L$ triplets. This mechanism is called see-saw mechanism.

Type I see-saw: extra fermion singlets

The first model is to add new heavy sterile right-handed neutrinos to the theory. They can have both a Yukawa interaction and a Majorana mass term.

$$L = L_{SM} - \frac{1}{2}\bar{N}_i i\partial\!\!\!/ N_i + (-\frac{1}{2}M_N^{ij}\bar{N}_i N_j - \lambda_N^{ij}\bar{L}_i\tilde{\phi}N_j + h.c.)$$

So the neutrinos have a 6×6 Majorana/Dirac mass matrix:

$$\begin{array}{cc} & \begin{array}{cc} \nu_L & N \end{array} \\ \begin{array}{c} \nu_L \\ N \end{array} & \begin{pmatrix} 0 & \lambda_N^T v \\ \lambda_N v & M_N \end{pmatrix} \end{array}$$

In practise we do not know the values of λ_N and M_N but we can still consider two interesting limits:

1. $M_N \gg \lambda_N$: In this case we will have 3 almost pure right-handed neutrinos with heavy Majorana masses M_N and 3 almost pure left-handed neutrinos with light Majorana mass $m_\nu = -(v\lambda_N)^T M_N^{-1}(v\lambda_N)$. Now Integrating out the heavy neutrinos gives the following non-renormalizable effective Lagrangian:

$$L_{eff} = -(\lambda_N^T M_N^{-1} \lambda_N)_{ij} \tilde{\phi}_\alpha \bar{L}_i{}^\alpha L_j^\beta \tilde{\phi}_\beta$$

2. $M_N \ll \lambda_N$: In this case we have 3 Dirac neutrinos $\Psi = (\nu_L, \bar{N})$ with mass $m_\nu = \lambda_N v$. As we saw earlier, this means that in order to get the observed neutrino masses one needs $\lambda_N \sim 10^{-12}$.

As we saw, since Majorana masses arises naturally but one needs to force the theory to have Dirac masses (impose the conservation of lepton number and setting M_N to zero), the Majorana neutrinos are considered as more likely.

Type III see-saw: extra fermion triplets

We can instead add extra fermion N which is a triplet under $SU_L(2)$ with zero hypercharge. One can show that as long as $M_N \gg v$ everything is the same as type I: triplet exchange generates the Majorana mass operator $\tilde{\phi}_\alpha \bar{L}^\alpha L^\beta \tilde{\phi}_\beta$.

Type II see-saw: extra scalar triplet

One can also add a new complex Higgs scalar, T, which carries hypercharge -1, is a colour singlet and a triplet under $SU_L(2)$. Now after writing the most generic renormalizable Lagrangian and integrating out the heavy triplet one can generate the Majorana mass operator.

2.3 the PMNS mixing matrix

We extend the SM by adding to its Lagrangian the non-renormalizable operator $\tilde{\phi}_\alpha \bar{L}^\alpha L^\beta \tilde{\phi}_\beta$ and no new fields. This operator is going to give the Majorana neutrino masses. We can diagonalize the neutrino mass matrix by redefining the neutrino fields:

$$\nu_l = V_{li} \nu_i$$

where V is a unitary matrix which is very similar to CKM mixing matrix for quarks. This mixing matrix may be parameterized in terms of mixing angles and phases such that $V = UK$ with $K = Diag(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ and

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

In here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ where θ_{ij}'s are the mixing angles. The quantities δ, α_1 and α_2 are CP-violating phases. If the neutrinos are Dirac fields we can rotate them to remove the phases α_i, that is why they are also called the Majorana phases. The CKM-like matrix, U_{ai}, is called the PMNS matrix for Pontecorvo, Maki, Nakagawa and Sakata.

3 Neutrino oscillation

Neutrinos are usually created by charged-current weak interaction. This means that they are produced in association with a charged lepton and therefore they are created in flavour eigenstates. But the eigenstates of time evolution operator are mass eigenstates not the flavour eigenstates. So when neutrinos propagate and reach the detector, another charged-current reaction, the detector probes the final flavour state which might not be the same as initial state if the mass and flavour eigenstates are different. This is called neutrino oscillation.

3.1 Vacuum oscillation

Suppose a neutrino is produced in a flavour eigenstate, ν_a. It then propagates to a detector at distance L in time t, where it is measured to have flavour ν_b. The amplitude for this process is:

$$\begin{aligned} \langle \nu_b(L,t)|\nu_a(0.0)\rangle &= \langle \nu_b|exp(-i\hat{H}t + i\hat{P}.L)|\nu_a\rangle \\ &= \sum_i \sum_\sigma \int d^3k e^{-iE_i(k)t+ik.L} \langle \nu_b|\nu_i(k,\sigma)\rangle\langle \nu_i(k,\sigma)|\nu_a\rangle \end{aligned}$$

Here $\hat{H}$ and $\hat{P}$ are the time evolution and transition operators and we have inserted a complete basis of mass eigenstates. Because the neutrinos are almost massless we can assume $L = t$. Small deviation from $L = t$ leads to just a species-independent phase which is not important for neutrino oscillation. We can take E to be a known variable since in typical application the neutrino energy is measured accurately. Given E, we can use the approximation that neutrinos are ultra-relativistic and $E - |k| = \frac{m_i^2}{2E}$. Thus up to an irrelevant overall phase, the transition amplitude is

$$\begin{aligned} \langle \nu_b(L,t)|\nu_a(0.0)\rangle &= \sum_i e^{-im_i^2 L/(2E)} \langle \nu_b|\nu_i\rangle\langle \nu_i|\nu_a\rangle \\ &= \sum_i e^{-im_i^2 L/(2E)} V_{bi} V_{ai}^* \end{aligned}$$

And the probability for the oscillation ($\nu_a \to \nu_b$) is

$$P(\nu_a \to \nu_b) = \delta_{ab} - 4\sum_{i>j} Re(U_{bi}U^*_{bj}U_{a,j}U^*_{ai})\sin^2(\phi_{ij}) + 2\sum_{i>j} Im(U_{bi}U^*_{bj}U_{a,j}U^*_{ai})\sin(2\phi_{ij})$$

where $\phi_{ij} = \frac{\Delta m^2_{ij} L}{4E}$ and $\Delta m^2_{ij} = m^2_j - m^2_i$. Notice that in the above equation the Majorana phases, $e^{i\alpha_i}$, cancel each other. Therefore one can not determine if the neutrinos are Dirac or Majorana just based on the oscillation experiments.

Using the above equation we see that:

$$P(\nu_a \to \nu_b; U^*) = P(\nu_b \to \nu_a; U) = P(\bar{\nu_a} \to \bar{\nu_b}; U)$$

The second equality is correct if CPT invariance holds. Thus the probability for oscillation of an antineutrino is the same as that for a neutrino, if we replace the mixing matrix U with its complex conjugate. So if U is not real, the CP is violated: the neutrino and antineutrino oscillation probabilities are different.

The oscillation length is defined by

$$\lambda = \frac{4\pi E}{\Delta m^2} = 2.48km \frac{E}{GeV} \frac{eV^2}{\Delta m^2}$$

At the limit $L \ll \lambda$ the probability becomes diagonal (because the PMNS matrix is unitary). Thus in order to see the oscillation, one needs to increase the length between the source and the detector and decrease the energy of neutrinos.

If one squared-mass splitting is much bigger than the other ones, which is often the case, then we can consider the two-neutrino special case. The unitary matrix U_{ai} now can be chosen:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

In this case the oscillation probability is

$$\begin{aligned} P(\nu_a \to \nu_a) &\simeq 1 - \sin^2(2\theta)\sin^2(\frac{\Delta m^2 L}{4E}) \\ P(\nu_a \to \nu_b \neq \nu_a) &\simeq \sin^2(2\theta)\sin^2(\frac{\Delta m^2 L}{4E}) \end{aligned}$$

In a realistic setup, one needs to average over some energy range ΔE and some path-length range ΔL. In the limit that $L \gg \lambda$ this averaging leads to $\langle \sin^2(\frac{\Delta m^2 L}{4E}) \rangle = \frac{1}{2}$ and

$$P(\nu_a \to \nu_a) = 1 - \frac{1}{2}\sin^2(2\theta), \qquad P(\nu_a \to \nu_b \neq \nu_a) = \frac{1}{2}\sin^2(2\theta)$$

We could re-drive the above transition probability by combining the probabilities rather than amplitudes.

3.2 Oscillation in normal matter

Although the neutrino interactions are weak, the presence of matter can have significant effect on the way neutrinos propagate. These effects are somehow similar to the fact that the speed of light travelling through a transparent medium is changed by the index of reflection which is different fo different polarizations of light. The same thing happens for the neutrinos. Because the normal matter is composed by electron rather than by muon or tau, the ν_e interact differently than the ν_μ and ν_τ and so the refraction index would be flavour-dependant. This effect in known as MSW (Mikheyev Smirnov Wolfenstein) effect. Now suppose the neutrinos are propagating in an environment like the interior of the sun. Neutrinos can interact with other particles through both neutral- and charged-current interactions. In the neutral-current interaction, since scattering of ν_l on electrons and quarks is the same for all flavours, it shifts all the neutrino types by the same amount and thus it is not important in neutrino oscillation. On the other hand there is an interesting effect due to $\nu_e e$ scattering mediated by the W boson which is described by

$$L_{cc} = \frac{4G_F}{\sqrt{2}}[i\bar{\nu}_e\gamma_\mu P_L\nu_e][i\bar{e}\gamma_\mu P_L e]$$

But the $i\bar{e}\gamma_\mu e$ term is just the the electron current operator and its mean value in the medium rest frame is $\langle J_e^\mu \rangle = N_e\delta_0^\mu$ where N_e is the local electron number density. The $i\bar{e}\gamma_5 e$ term gives the axial electron current which vanishes for the parity-invariant environment. So the effective matter Hamiltonian density is

$$\langle H_{eff} \rangle = An\bar{u}_l\gamma_0 P_L\nu_l$$

where A is called matter potential.

$$A = \sqrt{2}G_F[N_e diag(1,0,0) + constant \times I]$$

Usually the neutrino index of refraction is so closed to 1 ($n - 1 \simeq \frac{A}{E_\nu} \ll 1$) that we can not observe the optical effects but since $\frac{A}{(\Delta m^2/E_\nu)}$ can be equal or even larger than one the presence of matter can significantly affect oscillation. Let's consider the case with two neutrinos again. If we add this effective MSW Hamiltonian to the Hamiltonian which describes the propagation of neutrinos in the vacuum and then drop all terms proportional to the unit matrix, we find the following effective mass matrix within the normal matter.

$$\delta H = \frac{G_F N_e}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + \frac{\Delta m^2}{4E}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

Which we can re-write in terms of an effective medium-dependent mass splitting and mixing angle:

$$\delta H = \Delta_m\begin{pmatrix}-\cos 2\theta_m & \sin 2\theta_m\\ \sin 2\theta_m & \cos 2\theta_m\end{pmatrix}$$

where

$$\Delta_m = [(\frac{G_F N_e}{\sqrt{2}})^2 - (\frac{G_F N_e}{\sqrt{2}})\frac{\Delta m^2 \cos 2\theta}{2E} + (\frac{\Delta m^2}{4E})^2]^{1/2}$$
$$\sin 2\theta_m = (\frac{\Delta m^2}{4E})\frac{\sin 2\theta}{\Delta_m}$$

Notice that if

$$\frac{G_F N_e}{\sqrt{2}} = \frac{\Delta m^2}{4E} \cos 2\theta$$

then the medium-dependent mixing becomes maximal ($\sin 2\theta_m = 1$) even if $\theta \ll 1$. This is called resonance.

Now let's study the effect of matter in propagation of solar neutrinos. The experimental data show that only two neutrino mass eigenstates are significantly involved in the evolution of solar neutrinos. ν_e is created at the core of sun. The probability of this neutrino to be produced in the heavier mass eigenstate is then $\sin^2\theta_m$ and the probability to be in the lighter state is $\cos^2\theta_m$. Since the oscillation wavelength is much smaller than the radius of sun, neutrinos propagate for many oscillation wavelengths and the phases average out :we have to combine probabilities rather than amplitudes. If the density inside the sun changes very slowly (adiabatic approximation) each neutrino mass eigenstate remains the same. When they arrive at earth, the appropriate mixing angle is the vacuum one and so the amplitude of the mass eigenstate to be measured as electron type is controlled by $\sin\theta$ and $\cos\theta$. Therefore

$$P(\nu_e \to \nu_e) = \sin^2\theta_m \sin^2\theta + \cos^2\theta_m \cos^2\theta$$

As we can see from this equation, for low energy neutrinos we have $\Delta m^2/(4E) \gg G_F N_e/\sqrt{2}$ at the solar center, so $\theta_m = \theta$ and the survival probability becomes $\sin^4\theta + \cos^4\theta = 1 - \sin^2 2\theta$. On the other hand for high energy neutrinos $G_F N_e/\sqrt{2} \gg \Delta m^2/(4E)$ at the center of sun, so $\theta_m \simeq \pi/2$ and the probability of observing ν_e becomes $\sin^2\theta$.

4 Experiments

There are several kind of experiments with many different detector technology which provide a good evidence that the neutrinos do change flavour in the nature. We discuss this evidence.

4.1 the atmospheric evidence

The atmospheric neutrinos are produced in the Earth's atmosphere by the cosmic rays. Primary cosmic rays hit the nuclei of air in the upper part of atmosphere and produce mostly pions. Charged pions then decay to μ and ν_μ. The muons decay in turn into $\mu^- \to e^- \bar{\nu_e} \nu_\mu$ and $\mu^+ \to e^+ \nu_e \bar{\nu_\mu}$. Therefore one expects to detect a $\nu_\mu : \nu_e$ ratio close to $2:1$, but this ratio is seen only for downward moving neutrinos. For the upward moving neutrinos (the ones coming from other side of the Earth) this ration is close to $1:1$.

One can argue that since the the flux of cosmic rays which lead to neutrinos with energies of order a few GeV is isotropic, these neutrinos are produced at the same rate all around the Earth. Thus the flux coming down from the Zenith angle θ_z must be equal to the flux coming up from angle $\pi - \theta_z$.

The data from the underground Super-Kamiolande (SK) detector are shown in Fig.1 . The crosses are the data and their errors, the thin lines are the best-fit oscillation expectation and the thick lines are the no-oscillation expectation. As you can see, for multi-GeV atmospheric muon neutrinos, the event distribution doesn't have the $\theta_z \Longleftrightarrow \pi - \theta_z$ symmetry, the observed ν_μ flux coming up from zenith angle $\pi - \theta_z$ is half that coming down from angle θ_z. This can be explained by the fact that upward-going muon neutrinos have to travel a longer distance to reach the detector, so they have more time and distance to oscillate way into another flavour.

Fig.1 also shows that the zenith-angle distribution of e events show no asymmetry and the data are compatible with no oscillation. thus we can say that nothing happens to ν_e and that ν_μ oscillate into ν_τ.

Since atmospheric muon neutrinos are oscillating into another flavour, one would expect that a fraction of muon neutrinos generated in accelerators should disappear when they reach a sufficiently distant detector. Both K2K and MINOS experiments have observed this disappearance. K2K and MINOS measure the ν_μ flux once in a detector near the source before any oscillation has happened and once in a detector far away from the source (250 km in the case of K2K and 735 km in the case of MINOS). The results from these two experiments are consistent with two-neutrino oscillation.

All these observations point to the parameter

$$\sin^2 2\theta_{atm} = 1.02(4) \qquad (36^\circ < \theta_{atm} < 54^\circ, 99\% CL)$$

$$\Delta m^2_{atm} = |m_3^2 - m_2^2| = 2.5(3) \times 10^{-3} eV^2 \qquad (1.7 - 3.3, 99\% CL)$$

and since we saw that ν_μ almost entirely oscillate to ν_τ then

$$\theta_{atm} = \theta_{23} \qquad \theta_{13} \simeq 0$$

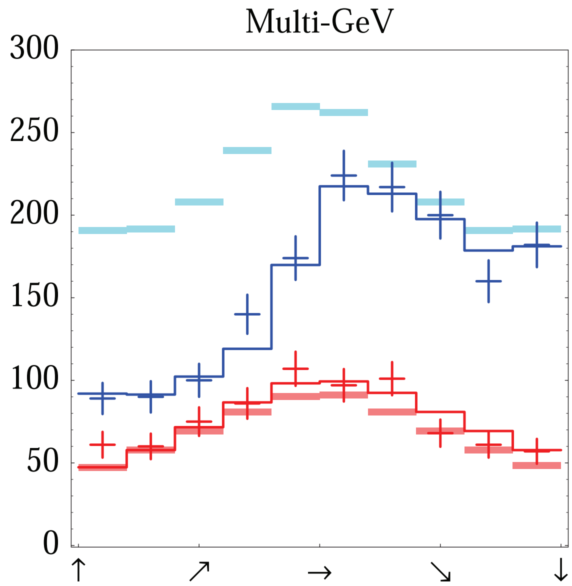

Figure 1: SK atmospheric data for multi-GeV neutrinos: Number of e^{+-}(red) and of μ^{+-}(blue) events as a function of direction of scattered lepton.(From A. Strumia and F. Vissani, arXiv:hep-ph/0606054v3)

4.2 The solar evidence

Solar neutrino experiments were the first evidence for neutrino oscillations. The nuclear processes in the sun produce only ν_e, the experiments have shown that the solar ν_e flux arriving at the Earth is below the one expected from neutrino production calculations. Around the center of sun the energies are produced through the following reaction:

$$4p + 2e \longrightarrow {}^4\text{He} + 2\nu_e$$

This reaction proceeds in a sequence of steps which are summarized in Fig.2 .

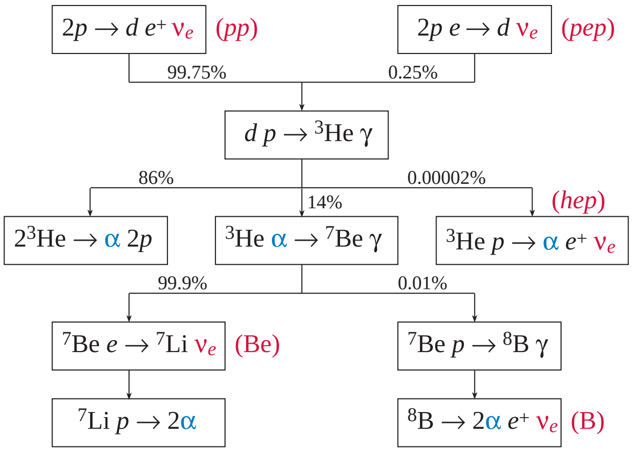

Figure 2: $4p + 2e \rightarrow {}^4\text{He} + 2\nu_e$ chain inside the sun. (From A. Strumia and F. Vissani, arXiv:hep-ph/0606054v3)

Sudbury Neutrino Observatory (SNO) detects the high-energy solar neutrinos from B decay via the following reactions

$$\nu + d \rightarrow e^- + p + p$$

$$\nu + d \rightarrow \nu + p + n$$

Only ν_e participates in the first reaction, thus it measures the flux $\phi(\nu_e)$ of ν_e. In the second reaction, all the three flavour neutrinos can participate, so it measures $\phi(\nu_e) + \phi(\nu_{\mu,\tau})$. According to SNO:

$$\frac{\phi(\nu_e)}{\pi(\nu_e) + \phi(\nu_{\mu,\tau})} = 0.340 \pm 0.023(stat)\ {}^{+0.029}_{-0.031}(syst)$$

This shows that $\phi(\nu_{\mu,\tau})$ is not zero and some of ν_e produced in the sun do change flavour. This behaviour implies that a fraction of $\bar{\nu_e}$ generated in a reactor disappear into the antineutrinos of another flavour if they travel more than a hundred kilometres to reach the detector. The KamLAND experiment confirms this disappearance. In this experiment, $\bar{\nu_e}$ typically travel 180 km to reach the detector. The result of KamLAND is consistent with two-neutrino oscillation. Since atmospheric oscillations have negligible effect on solar neutrinos and KamLAND data ($\nu_\mu \rightarrow \nu\tau$ oscillation is unimportant for these experiments) and the mixing angle θ_{13} is so small, we can conclude that the solar $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillation depends only on two parameters:

$$\Delta m^2_{sun} = \Delta m^2_{12} \qquad\qquad \theta_{sun} = \theta_{12}$$

The numerical value of these parameters are found by experiments to be:

$$0.40 \leq \tan^2\theta_{sun} \leq 0.50 \qquad (30^\circ < \sin\theta_{sun} < 38^\circ, 99\%CL)$$
$$\Delta m^2_{sun} = 8.0(3) \times 10^{-5} eV^2 \qquad (7.1 - 8.9, 99\%CL)$$

5 Questions to be answered in the future

Even though now we have strong evidence for neutrino oscillation, there are still many open questions to be answered by future experiments. we address a few of them.

1. **Are the neutrino mass eigenstates Majorana or Dirac?**

   As we saw oscillation experiments are insensitive to the Majorana phases α_1 and α_2 and therefore they can not determine whether the neutrinos are Majorana or Dirac. Can we confirm this experimentally? The promising approach is to search for neutrino-less double beta decay($0\nu\beta\beta$).

   In double beta decay, two neutrons in the nucleus are converted into two protons and emit two electrons and two electron antineutrinos. If the neutrinos where Majorana (antineutrino and neutrino are the same particle) then the final two antineutrinos in double beta decay can annihilate. This process is forbidden in the SM model since the lepton number is violated. The decay rate of the process is equal to $\Gamma = G|M|^2|m_{\beta\beta}|^2$ where G is just the phase-space factor and $|M|^2$ is the matrix element, and $m_{\beta\beta}$ is the so-called effective Majorana neutrino mass.

   $$|m_{\beta\beta}| = |\sum_i m_i U^2_{ei}|$$

   The discovery of $0\nu\beta\beta$ proves that the neutrinos are Majorana. Right now the best limit on the decay lifetime comes from ${}^{76}_{32}Ge \to {}^{76}_{34}S_e + 2e^-$ process (1.9×10^{25} years) which leads to constraint $|m_{\beta\beta}| < 0.35eV$.

2. **Is the mass pattern a normal hierarchy or an inverted one?**

   As we discussed in the previous section, from oscillation experiments we found:

   $$\Delta m^2_{atm} = |\Delta m^2_{23}| \ > \ \Delta m^2_{sun} = |\Delta m^2_{12}|$$

   But neutrino oscillations do not fix the absolute value of each of the masses separately and leave us two possibilities: a positive Δm^2_{23} or normal hierarchy which means that the nearly degenerated pair ν_1 and ν_2 are less massive than ν_3. The other possibility is that $m_1, m_2 > m_3$ which is called the inverted hierarchy. We can find which of these two case is correct by finding the mass of each neutrino.

3. **What are the masses of neutrino mass eigenstates?**

   Oscillation experiments just tell us about the difference of squares of masses, not the distance of the entire pattern from zero. We can use beta decay or neutrino-less double beta decay to learn about the masses of mass eigenstates. Also one can obtain some information from cosmology or astrophysics. Cosmological data roughly probe the sum of neutrino masses and the current limit is $\sum_i m_i > (0.17 - 2.0eV)$ depending on the cosmological assumptions.

4. **Can we observe a CP violation behaviour by studying neutrinos?**

   We saw that from data there is a bound on θ_{13} which is $s_{13}^2 < 0.032$. By looking at the PMNS mixing matrix, we find that the CP-violating phase δ only enters the U in combination with s_{13}. Thus the size of CP violation in oscillation depends on s_{13}. The CHOOZ and DOUBLECHOOZ experiments are going to give us a better bound on this mixing angle θ_{13}. The baryon asymmetry of the universe indicates that some CP violation had to occurred during the early universal. Since the known source of CP violation in the quark sector is not large enough, some sort of leptonic CP violation must be responsible for this baryon asymmetry.

To answer these questions, we need to have progress both in the future oscillation experiments (such as solar, atmospheric and reactor experiments) and future non-oscillation experiments (such as β decay, neutrino-less double beta decay experiments and cosmological observations).

References

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