PHYS 526 Homework #10

Due: Nov. 19, 2013

0. Read Chs. 7.4, 9 of Peskin and Schroeder and `notes-09`, `notes-10`.

1. Draw the leading-order Feynman diagrams for the process $\gamma\gamma \to \gamma\gamma$, in QED and work out the corresponding constributions to the scattering matrix element.

2. Find the total unpolarized cross section for muon-positron elastic scattering at leading order in the center-of-mass frame. You may assume that the collision energy is very high and neglect the electron and muon masses.

3. Compute the leading-order matrix element for electron-positron annihilation into a pair of photons. Using this, find the summed and squared matrix element that you would plug into the expression for the total unpolarized cross section. You can leave your expression in terms of a trace over Dirac matrices, but there should be no explicit polarization vectors left over.

4. The Feynman rules for a massive vector boson are very similar to those of the photon, but with two important differences. The first is that the propagator is

$$\frac{i}{p^2 - M^2}\left(-\eta^{\mu\nu} + p^\mu p^\nu/M^2\right) .$$

   The second is that there are now three polarizations (since a massive vector creates states with $s = 1$), and they have the completeness relation

$$\sum_{\lambda=1}^{3} \epsilon^{\mu *}(p,\lambda)\, \epsilon^{\nu}(p,\lambda) = -\eta^{\mu\nu} + p^\mu p^\nu/M^2 .$$

   Suppose a massive vector Z^μ couples to the Dirac fermion Ψ according to

$$-\mathscr{L}_{int} = g\, Z_\mu\, \bar{\Psi}\gamma^\mu P_L \Psi .$$

   Compute the total unpolarized decay rate for $Z \to \Psi\bar{\Psi}$ in this theory. You may neglect the mass of Ψ relative to the mass of Z.
   Hint: use $\gamma^\mu P_L = P_R \gamma^\mu$, $P_{L,R} = (1 \mp \gamma^5)/2$, and employ the trace relations for γ^5.

5. Connections.
   Consider a fermion Ψ in a theory with a $U(1)$ gauge invariance (generalizing what happens in QED) under $\Psi(x) \to e^{-i\alpha(x)}\Psi(x)$. Suppose there also exists an object $P(x,y)$ that transforms under gauge as $P(x,y) \to e^{-i\alpha(x)} P(x,y) e^{i\alpha(y)}$ with $P(x,x) = 1$.

   a) How does $P(x,y)\Psi(y)$ transform?

   b) Recall the basic definition of a derivative: $df/dx = \lim_{\epsilon\to 0}[f(x+\epsilon) - f(x)]/\epsilon$. This means that a regular derivative on $\Psi(x)$ doesn't make much sense, since it involves the difference between objects with different gauge transformation properties. A more logical definition is the operator $\tilde{D}_\mu$, given by

$$n^\mu \tilde{D}_\mu \Psi(x) = \lim_{\epsilon \to 0} \left[P(x, x+\epsilon n)\Psi(x+\epsilon n) - \Psi(x) \right] / \epsilon \ .$$

where n^μ is any unit 4-vector. Evaluate the action of this operator on $\Psi(x)$ by expanding $P(x, x+\epsilon n) = [1 + i\epsilon n_\mu \tilde{A}^\mu(x) + \mathcal{O}(\epsilon^2)]$, and writing the result in terms of regular derivatives and $\tilde{A}^\mu(x)$. Does it remind you of anything?

c) How does $n^\mu \tilde{D}_\mu \Psi(x)$ transform under gauge?

d) The quantity $\tilde{A}^\mu(x)$ is called a *connection.* Since $P(x, y)$ transforms under gauge, $\tilde{A}^\mu(x)$ must as well. Work out how it does by applying a gauge transformation to $P(x, x+\epsilon n)$, expanding to linear order in ϵ, and matching up powers of ϵ. Again, does this remind you anything?