PHYS 526 Homework #9

Due: Nov. 12, 2013

0. Read Ch.5 of Peskin & Schroeder and `notes-08`.

1. Work out the explicit details of Eqs. (19–28) in `notes-08`.

2. Dimensional Analysis.

   a) In natural units ($\hbar = c = 1$), the action has to be dimensionless. Based on the structure of its kinetic term, what is the mass dimension of the scalar field operator in four spacetime dimensions?

   b) What is the mass dimension of the scalar field in d spacetime dimensions?

   c) What is the mass dimension its mass parameter M in d spacetime dimensions?

   d) What are the mass dimensions of the Dirac fermion and photon fields in d spacetime dimensions?

   e) What is the mass dimension of the coupling y in QED lite and the electromagnetic coupling e in QED in d dimensions? Is there anything special about $d = 4$?

   Hint: the Lagrangians for these theories are given in `notes-09`.
   Hint: for $d \neq 4$, *assume the Lagrangian density has the same functional form as* $d = 4$.

3. Consider a theory with a real scalar and two Dirac fermions:

$$\mathscr{L} = \bar{\Psi}_1(i\not{\partial} - m_1)\Psi_1 + \bar{\Psi}_2(i\not{\partial} - m_2)\Psi_2 + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}M^2\,\phi^2 - y_1\phi\bar{\Psi}_1\Psi_1 - y_2\,\phi\bar{\Psi}_2\Psi_2\ .$$

   Compute the scattering matrix element at leading order for $\Psi_1\bar{\Psi}_1 \to \Psi_2\bar{\Psi}_2$ in the center-of-mass frame.

4. Consider instead a slightly different theory with two Dirac fermions but no scalar:

$$\mathscr{L} = \bar{\Psi}_1(i\not{\partial} - m_1)\Psi_1 + \bar{\Psi}_2(i\not{\partial} - m_2)\Psi_2 - \frac{1}{f^2}(\bar{\Psi}_1\Psi_1)(\bar{\Psi}_2\Psi_2)\ .$$

   a) What is the appropriate momentum-space Feynman rule for the new interaction. And what is the mass dimension of the new coupling f (in $d = 4$ spacetime dimensions)?

   b) Compute the scattering matrix element at leading order for $\Psi_1\bar{\Psi}_1 \to \Psi_2\bar{\Psi}_2$ in the center-of-mass frame.

   c) Show that there is a special value of f for which the scattering matrix element computed in this theory matches the one for the theory in #3 in the simultaneous limits of $s \ll f^2$ and $s \ll M^2$ (where $s = (p_1 + p_2)^2$). For this reason, the theory of #4 is said to be the low-energy *effective field theory* obtained from #3 in the limit that the energy is much less than the mass of the scalar.

5. More Dirac fun.

a) Show that $\gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$.

b) Use this to find the complex conjugates of the following:

   i) $\bar{u}(p)\gamma^\mu v(k)$
   ii) $\bar{u}(p)\gamma^\mu P_L u(k)$
   iii) $\bar{u}(p)\gamma^5 u(k)$
   iv) $\bar{v}(p)\gamma^\mu\gamma^5 v(k)$
   v) $\bar{v}(p)\gamma^\mu\gamma^\nu v(k)$

c) Compute the summed and squared matrix element for $\Psi(p_1)\,\phi(p_2) \to \Psi(p_3)\,\phi(p_4)$ in QED lite. By this, we mean "$|\mathcal{M}|^{2}$" $= (1/2)\sum_{s_1,s_3}|\mathcal{M}|^2$, the quantity that would be integrated over phase space to get the unpolarized total cross sections. Use the completeness relations for Dirac spin polarizations to eliminate the spin sums and evaluate the resulting traces. To simplify your life, you can take $m \to 0$.