PHYS 528 Lecture Notes #8

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1 Dealing with Quantum Corrections

Most of the calculations in this course will be strictly at tree-level. This aproximation gives a good qualitative description of the electromagnetic and weak interactions, but it fails very badly for the strong force. Before moving on to discuss the theory of the strong force, QCD, we will therefore spend a bit of time discussing how to think about loop corrections in a quantum field theory. [1] By expanding in powers of $\hbar$, one can show that these correspond to quantum corrections to the field theory [1].

To illustrate our arguments, we will focus on the specific and relatively simple field theory defined by the Lagrangian

$$\mathscr{L} = \frac{1}{2}Z_0(\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2 - \frac{\lambda_0}{4!}\phi^4. \tag{1}$$

Even though we choose this specific theory, our arguments will generalize (possibly with some minor complications) to other more general theories.

Loop corrections can become slightly complicated because they seem to involve ultraviolet divergences. These arise from summing over all intermediate virtual momentum states, including very high momentum modes, that contribute to the quantum mechanical amplitude [2, 3]. The presence of infinities certainly looks bad, but they don't have to spoil the predictivity of the quantum field theory. The modern point of view is that these apparent infinities just imply that the theory we started with does not work all the way up to arbitrarily high energies. Instead, some other theory should kick in. In the Standard Model (SM), for example, we expect quantum gravity to become important at $E \sim M_{\rm Pl}$. Thus, as relatively low-energy observers, we should not take the apparent divergences in loop amplitudes too seriously. Rather, we should identify them with *finite* but unknown contributions from unknown UV physics.

Trading infintity for finite-but-unknown may not sound like much, but at least it gives us license to manipulate the would-be divergences. In a *renormalizable* field theory there are finitely many, N say, fundamental "divergences". This means that we can fix the values of the N unknown quantities by making N observations and comparing them to the predictions of the theory. Once we have done this, we can make unambiguous predictions for all other observables in the theory.

In the sample theory of Eq.(1) there are three fundamental divergences. We will regulate them schematically by imposing a large-momentum cutoff Λ that is very large compared to

[1]Everything in this section is highly schematic. To get a more accurate story, you should take a full course in QFT and read the texts listed among the references.

the energies we are testing the theory at. By dimensional analysis, the structure of quantum corrections is

$$\begin{aligned}
&\text{2-point}: \quad p^2\left[A_1\ln\left(\frac{\Lambda^2}{a_p p^2+a_m m^2}\right)+A_2\right]-\left[B_1\Lambda^2+B_2\ln\left(\frac{\Lambda^2}{b_p p^2+b_m m^2}\right)+B_3\right] \\
&\text{4-point}: \quad C_1\ln\left(\frac{\Lambda^2}{c_p p^2+c_m m^2}\right)+C_2
\end{aligned} \tag{2}$$

The coefficients of all three corrections are functions of the coupling λ_0 and must vanish in the limit $\lambda_0 \to 0$, where the theory reduces to a free scalar theory.

An obvious observable in the theory of Eq. (1) is the cross-section $\sigma(\phi\phi \to \phi\phi)$. The amplitude (at one-loop order) is given by

$$-i\mathcal{M} = -i\lambda - i\left[C_1\ln\left(\frac{\Lambda^2}{c_p p^2+c_m m^2}\right)+C_2\right] \tag{3}$$

At one-loop order, both C_1 and C_2 are proportional to λ_0^2. Squaring this gives the cross-section. We can fix the value of λ_0 by measuring this cross-section at the CM energy $s = p^2$, and also fix the mass and field normaliztion by measuring the physical mass value. This then allows us to predict the cross-section at different energies. We find (schematically)

$$\sigma(p'^2) = \sigma(p^2) + 2\tilde{C}_1\ln\left(\frac{\tilde{c}_p p^2+\tilde{c}_m m^2}{\tilde{c}_p p'^2+\tilde{c}_m m^2}\right). \tag{4}$$

All the dependence on the unkown UV physics has cancelled out when we write an observable in terms of the other reference observables. Note that the coefficients here are slightly different from those in the vertex correction since we also have take into account wave function factors in computing the cross-section. However, all the coefficients here are calculable order-by-order in perturbation theory.

It helps to be a bit more systematic about dealing with the ersatz divergences. For this, it is often useful to rewrite things in terms of an *effective action* with finite couplings [2, 3]. For this, let us also define new variables according to

$$Z_0 = (Z - \delta Z), \quad m_0^2 = (m^2 - \delta m^2), \quad \lambda_0 = (\lambda - \delta\lambda). \tag{5}$$

The idea is to now compute with Z, m^2, and λ, and treat the *counterterm* δ's as higher-order corrections that we use to absorb the dependence on the cutoff Λ. For example, the 4-point function becomes

$$\text{4-point} = -i\lambda - i\left[C_1\ln\left(\frac{\Lambda^2}{c_p p^2+c_m m^2}\right)+C_2\right] + i\delta\lambda, \tag{6}$$

with the calculable coefficients such as C_1 now being functions of λ rather than λ_0. Thus, we can remove the Λ dependence by choosing

$$\delta\lambda = C_1\ln\left(\frac{\Lambda^2}{\mu^2}\right), \tag{7}$$

where μ is an unspecified *renormalization scale* with dimensions of mass. It is needed to make the argument of the logarithm dimensionless. With this choice, the 4-point function becomes

$$-i\left[\lambda + C_1 \ln\left(\frac{\mu^2}{c_p p^2 + c_m m^2}\right) + C_2\right] \tag{8}$$

There is now no explicit dependence on the UV cutoff Λ, but we do have a new dependence on the unspecified parameter μ. (Recall as well that the coefficients are now functions of λ rather than λ_0.) We can similarly remove the dependence on Λ in the 2-point function by taking

$$\delta Z = A_1 \ln\left(\frac{\Lambda^2}{\mu^2}\right), \quad \delta m^2 = B_1\Lambda^2 + B_2 \ln\left(\frac{\Lambda^2}{\mu^2}\right). \tag{9}$$

The procedure described above, of splitting the bare Lagrangian parameters into finite couplings and counterterms, is called *renormalized perturbation theory* [3]. We have effectively hidden the unknown UV dependence within the finite parameters λ, m^2, and Z. These parameters are just a useful set of finite intermediate variables with which to express the quantum-corrected n-point functions. In particular, they are not physical since they depend on the unphysical renormalization scale μ and the choice of counterterms implicit in their definition is not unique.[2] To see the implicit dependence of λ, m^2, and Z on μ, notice that the original Lagrangian parameters are μ-independent, so the dependence on μ between the couplings and the counterterms must match via Eq. (5).

So how do we make sense of this result? Just like before, to obtain predictions for observables we must take three input observables to fix the values of λ, Z, and m^2 (for some spcified value of μ^2). Having done this, we can go and make predictions for other observables. These observables must be independent of μ. This is nearly identical to what we did previously to get rid of the cutoff dependence by always writing observables in terms of other observables. However, this change of parameters to the renormalized (but μ-dependent) set we are promoting turns out to be very useful. We can choose μ to be whatever value we like. In particular, when making predictions for observables at momenta $|p^2| \gg m^2$, the choice of $\mu^2 \sim p^2$ minimizes the logarithmic correction in Eq. (8). This improves the convergence of the perturbative expansion, and lets us identify $\lambda(\mu^2 \sim p^2)$ with the physical scattering amplitude up to rescaling by Z and small perturbative corrections. Put another way, a good choice of μ^2 makes the parameters we are using "close" to what we would identify as the physical couplings and masses. On the other hand, setting μ^2 very different from p^2 can lead to a poor convergence of the perturbative expansion, even for small $\lambda(\mu)$, due to a large logarithmic enhancement of the coefficients of the pertubative expansion.

These properties motivate us to put together a renormalized *effective action* $\Gamma[\varphi]$:[3]

$$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2}Z(\mu)(\partial\varphi)^2 - \frac{1}{2}m^2(\mu)\varphi^2 - \frac{\lambda(\mu)}{4!}\varphi^4 + (\text{non-local terms})\right]. \tag{10}$$

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[2]The choice made here, where we only remove the divergence and nothing else, is called *minimal subtraction.* Another choice would be to also cancel off some of the finite pieces such as C_2.

[3]The full definition is $\Gamma[\varphi] = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\int \prod_{i=1}^{n} d^4x_i\ \varphi(x_i)\right]\ \Gamma^{(n)}(x_1, \ldots, x_n)$, where $\Gamma^{(n)}$ is the n-point truncated 1PI renormalized Green's function. Note that this full object is highly non-local [2].

The point of the effective action is that if you compute with it at *tree-level*, you get the full quantum-corrected n-point 1PI truncated Green's functions. The non-local terms in this expression generate the explicitly p^2-dependent parts of the full quantum-corrected n-point functions discussed above. The non-local terms will be relatively small provided λ is small and we choose μ in a clever way. For processes characterized by the momentum scale p^2, the clever choice is $\mu \sim \sqrt{|p^2|}$. In this case the quantities

$$\tilde{\lambda}(\mu) = \lambda/Z^2, \qquad \tilde{m}^2(\mu) = m^2/Z, \tag{11}$$

will be "close" to the physical coupling and mass.

If we understand a theory at one energy scale, we can extrapolate its behaviour to other energy scales. A reasonable approximation to this can be had from figuring out how λ and m^2 shift with changing μ. The evolution of these couplings with μ goes by the name of the *renormalization group.* Let us define

$$\gamma \equiv -\frac{\mu}{Z}\frac{dZ}{d\mu} = -\frac{d\ln Z}{d\ln\mu} \tag{12}$$

$$\beta_\lambda \equiv \frac{d(\lambda/Z^2)}{d\ln\mu} \tag{13}$$

For the example given above, we have $0 = d(Z - \delta Z)/d\mu$, which yields

$$\gamma = 2A_1. \tag{14}$$

Similarly,

$$\beta_\lambda = -2C_1 + 4\tilde{\lambda}A_1, \tag{15}$$

and

$$\frac{d\tilde{m}^2}{d\ln\mu} = -2B_2 + 2\tilde{m}^2 A_1. \tag{16}$$

These are differential equations in $\tilde{\lambda}$ and $\tilde{m}^2$. By solving them we can extrapolate the theory to different energy scales without running into dangerously large logarithms.

Before moving on, let us mention a second variety of effective action. What we have described above is the 1PI effective action, and it is a generator of 1PI matrix elements that include quantum corrections from all energy scales. There is also the *Wilsonian* effective action that only has corrections from high-energy physics built-into it [2]. The Wilsonian effective action consists of an action together with an explicit UV cutoff. We write

$$S_{eff}(\Lambda) = \int d^4x \, \mathscr{L}_{eff}(\Lambda). \tag{17}$$

When computing with this action, it is understood that any would-be divergences are to be cut off at the scale Λ. The effective Lagrangian contains the usual kinetic, mass, and interaction terms, but it can (and usually does) also contain higher dimensional operators suppressed by powers of Λ. For example, in a real scalar theory one might have

$$-\mathscr{L}_{eff} \supset \frac{c_1}{\Lambda^2}(\partial\phi)^3 + \frac{c_2}{\Lambda^2}\phi^6 + \ldots \tag{18}$$

In general, all possible terms consistent with the symmetries of the theory can appear. This leads to a non-renormalizable theory, and we will describe it can still be predicitive a bit later. The interesting thing to look at with the Wilsonian effective action is how the coefficients of the operators in the theory change as we lower the cutoff Λ. Physically, this corresponds to sequentially *integrating out* the effects of high-energy physics. This gives a slightly different realization of the renormalization group compared to what we had earlier.

2 Anomalies

A surprising result of quantizing certain field theories is that quantum corrections sometimes explicitly break a symmetry of the classical Lagrangian. When this happens, the symmetry is said to be *anomalous* and the theory is said to have an anomaly [2]. Anomalies are an interesting and important feature of quantum field theories, and it would be easy to spend a whole course discussing them. Due to lack of time, we will only cover a few of the essential aspects of anomalies as they relate to the SM.

In formulating a quantum field theory, one typically needs both a Lagrangian to define the interactions and a procedure for regularization and renormalization to deal with apparent infinities. Anomalies arise when it is not possible to regularize/renormalize the theory in a way to preserves a classical symmetry of the Lagrangian. The presence of an anomaly can be deduced by computing the quantum expectation value of an operator containing the divergence of the classical Noether current:

$$\langle \mathcal{O}\, \partial_\mu j^\mu \rangle \neq 0, \tag{19}$$

where $\mathcal{O}$ is some operator in the theory.

Chiral fermions can be a source of anomalies in four dimensions. Consider a theory with left-handed fermions ψ_{L_i} and right-handed fermions ψ_{R_j}, a global symmetry G, and an Abelian gauge symmetry H. Let's assume the LH fermions have charges $Q^G_{L_i}$ and $Q^H_{L_j}$ under these groups and the RH fermions have charges $Q^G_{R_j}$ and $Q^H_{R_j}$. The Noether current for the global symmetry G is

$$j^G_\mu = \sum_i Q^G_{L_i} \bar{\psi}_{L_i} \gamma^\mu \psi_{L_i} + \sum_j Q^G_{R_j} \bar{\psi}_{R_j} \gamma^\mu \psi_{R_j}. \tag{20}$$

One can show that no matter how one regularizes the theory (in a Lorentz-invariant way), the expectation of the divergence of this current with a pair of gauge bosons is non-zero up to an overall coefficient equal to

$$\langle A_\mu A_\nu\, \partial^\lambda j^G_\lambda \rangle \;\propto\; \sum_i (Q^H_{L_i})^2 Q^G_{L_i} - \sum_j (Q^H_{R_j})^2 Q^G_{R_j}\ . \tag{21}$$

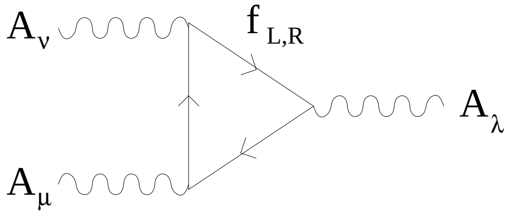

Figure 1: Fermion triangle diagram contributing to a gauge theory anomaly.

Unless this combination of charges vanishes, the expectation value is non-zero and the global symmetry G is anomalous. Note that it does vanish automatically if the theory is non-chiral, with all the LH and RH fermions coming in pairs with equal charges. Let us also mention that if we were to write all the SM fermion reps in terms of 2-component LH spinors there would be no pesky relative minus sign in Eq. (21). It is straightforward to generalize this result to non-Abelian symmetries, and we will do so below.

An anomaly in a global symmetry leads to interesting physical effects in the theory. However, an anomaly in a gauge symmetry would be disastrous since it would lead to a distinction between field configurations that are supposed to be physically equivalent. Therefore an important consistency condition for gauge theories with chiral fermions is that the gauge symmetries (treated as classical global symmetries) be anomaly-free. A sufficient condition for this to occur is that the sum of all fermion-loop triangle diagrams with three external gauge boson legs vanish – see Fig. 1. These diagrams are proportional to anomaly coefficients which depend on the chiral fermion representations involved.

For the SM, the non-trivially vanishing anomaly coefficients for all the possible anomalies in the theory are:

$$SU(3)_c^3 \propto \sum_L tr(t_c^a t_c^b t_c^c) - (L \to R) \tag{22}$$

$$SU(3)_c^2 \times U(1)_Y \propto \sum_L tr(t_c^a t_c^b Y) - (L \to R) \tag{23}$$

$$SU(2)_L^3 \propto \sum_L tr_L(t_L^p t_L^q t_L^r) - (L \to R) \tag{24}$$

$$SU(2)_L^2 \times U(1)_Y \propto \sum_L tr_L(t_L^p t_L^q Y) - (L \to R) \tag{25}$$

$$U(1)_Y^3 \propto \sum_L Y^3 - (L \to R) \tag{26}$$

$$(grav)^2 U(1)_Y \propto \sum_L Y - (L \to R) \tag{27}$$

Here, the sum $\sum_L$ runs over all left-handed fermion reps, and similarly for R. Sometimes you will see $\sum_L(...) = tr_L(...)$. These anomaly coefficients are just the group theoretic factors associated with the corresponding triangle loops weighted by a relative factor of minus one for chirality. Note that mixed anomalies with a single non-Abelian factor like $SU(3)_c^2 SU(2)_L$ or $SU(2)_L U(1)_Y$ vanish automatically since they all involve the trace of a single non-Abelian generator. The last condition is only needed if we want to eventually couple the theory in a consistent way to gravity (which we do).

e.g. 1. $SU(3)_c^2 U(1)_Y$ anomaly cancellation in the SM.

The $SU(3)_c \times U(1)_Y$ anomaly coefficient is

$$A_{331} = tr_L(t_c^a t_c^b Y) - (L \to R) \tag{28}$$

The L part of the trace gets contributions from Q. For this, there are two **3** reps of $SU(3)_c$, one each for the two $SU(2)_L$ components u_L and d_L, and both have hypercharge $Y = 1/6$. The R part of the trace comes from u_R and d_R which are both $\bar{\mathbf{3}}$ reps of $SU(3)_c$ and have hypercharges $Y = 2/3$ and $-1/3$. Putting things together, and using $tr(t_3^a t_3^b) = \delta^{ab}/2$, we find

$$A_{331} = n_g \left[\left(\frac{1}{2} \times 2 \times \frac{1}{6} \right) \quad - \quad \left(\frac{1}{2} \times \frac{2}{3} \quad - \quad \frac{1}{2} \times \frac{1}{3} \right) \right] = 0, \tag{29}$$

where $n_g = 3$ is the number of generations. This vanishes - Hooray! You will get to check that all the other potential SM gauge anomalies vanish in the homework.

3 Effective Field Theories

Sometimes less is more, and this is certainly true of quantum field theories. The modern view of quantum field theories, and the SM in particular, is that they are not fundamental theories of Nature [4]. Instead, we regard them as *effective field theories* that only provide an approximate description of how things work up to finitely large energies [5, 6, 7]. This point of view provides a new (and arguably more physical) perspective on the procedure of renormalization, and is often necessary to maintain a reliable perturbative expansion. It also frequently makes calculations much easier.

The main idea underlying EFT is that one should only keep around those degrees of freedom that can be produced on-shell given the range of energies one is studying. Correspondingly, the only dynamical fields in an EFT are those corresponding to on-shell particles. In this sense, an EFT has a built-in UV cutoff corresponding to energies large enough to produce new particles on shell. An EFT with massive particles also has a built-in IR cutoff at energies on the order of their masses. At energies below these masses, our EFT philosophy tells us to remove the massive particles to form an even simpler EFT.

All this stuff about EFTs will probably make more sense to you after seeing a specific example. A nice case is given by the weak interactions at relatively low energies, $E \ll m_W, m_Z$. Consider the decay of a muon by way of a W^- to $\nu_\mu \, e \bar{\nu}_e$. The amplitude for this is

$$\begin{aligned} -i\mathcal{M} &= \bar{u}_e(-i\frac{g}{\sqrt{2}}\gamma^\mu P_L)v_{\bar{\nu}_e} \, \bar{u}_{\nu_\mu}(-i\frac{g}{\sqrt{2}}\gamma^\nu P_L)u_\mu \, \frac{i}{p^2 - m_W^2} \left(-\eta_{\mu\nu} + p_\mu p_\nu / m_W^2\right) \\ &= -i\frac{g^2}{2m_W^2} \left(\frac{1}{1 - p^2/m_W^2} \right) \left(\eta_{\mu\nu} - p_\mu p_\nu / m_W^2\right) \bar{u}_e \gamma^\mu P_L v_{\bar{\nu}_e} \, \bar{u}_{\nu_\mu} \gamma^\nu P_L u_\mu \end{aligned} \tag{30}$$

Now, the momenta involved here are all less than the mass of the muon, which is a lot smaller than the mass of the W. Thus, we can expand this amplitude in powers of $p_\mu p_\nu / m_W^2$, and it is a good approximation (up to corrections of size p^2/m_W^2) to keep only the leading term. In this approximation, the amplitude becomes

$$-i\mathcal{M} \simeq i\frac{g^2}{2m_W^2}\ (\bar{u}_e\gamma^\mu P_L v_{\bar{\nu}_e})\ \left(\bar{u}_{\nu_\mu}\gamma_\mu P_L u_\mu\right). \tag{31}$$

It isn't hard to notice that exactly the same amplitude could have been obtained if we had started from a Lagrangian containing the interaction

$$\mathscr{L} \supset \frac{g^2}{2m_W^2}\,(\bar{e}\gamma_\mu P_L \nu_e)\ (\bar{\nu}_\mu\gamma^\mu P_L\mu). \tag{32}$$

Higher-order terms in the expansion could also be reproduced by including additional derivative operators.

This approximate equivalence motivates us to construct an EFT for the weak interactions at low energy which does not contain the $W^\pm$ or Z^0 vector bosons explicitly. Instead, we only keep the light fermions and include new interactions in the low-energy effective Lagrangian of the theory to account for the leading effects of the massive vectors, one of these being the operator of Eq. (32). Interactions mediated by exchanging a Z^0 would also give rise to operators such as

$$\frac{c_L^{qq}}{m_W^2}(\bar{q}\gamma^\mu P_L q)(\bar{q}'\gamma_\mu P_L q'), \quad \frac{c_R^{qq}}{m_W^2}(\bar{q}\gamma^\mu P_R q)(\bar{q}'\gamma_\mu P_R q'), \quad \ldots \tag{33}$$

The procedure of connecting a more complicated theory valid at high energies to the low-energy EFT is called *matching.* The matching given in Eqs. (32, 33) is accurate up to corrections on the order of p^2/m_W^2. The accuracy of the matching can be further improved to the order of $(p^2/m_W^2)^n$ $(n > 1)$ by including additional higher-derivative operators with even more powers of suppression by m_W^2 to reproduce the subsequent terms in the expansion of the W propagator. It should also be clear that the EFT ceases to be useful for larger momenta $p^2 \sim m_W^2$ because the expansion in powers of p^2/m_W^2 no longer converges quickly.

In general, we expect the matching procedure to produce every possible higher-dimensional operator consistent with the symmetries of the theory. If the full high-energy *ultraviolet completion* theory is known and is perturbative, as in the case of the electorweak interactions, we can do this matching explicitly in perturbation theory. More generally, a prescription for matching at the leading order in both the momentum and loop expansions is to simply replace the heavy fields that are being *integrated out* of the theory by the solutions of their classical equations of motion with all derivatives set to zero [6]. We illustrate this below.

e.g. 1. Integrating out a massive scalar or fermion.

Consider the theory

$$\mathscr{L} = |\partial\phi|^2 - m^2|\phi|^2 + \sum_{i=1}^{2}\bar{\psi}_i(i\gamma^\mu\partial_\mu - M_i)\psi_i - y\phi\bar{\psi}_1\psi_2 - y^*\phi^*\bar{\psi}_2\psi_1. \tag{34}$$

The classical equations of motion are:

$$\begin{aligned} 0 &= (\partial^2 + m^2)\phi + y\bar{\psi}_1\psi_2 \\ 0 &= i\gamma^\mu\partial_\mu\psi_1 - M_1\psi_1 - y\phi\psi_2 \\ 0 &= i\gamma^\mu\partial_\mu\psi_2 - M_2\psi_2 - y\phi^*\psi_1 \end{aligned} \tag{35}$$

We can look at the resulting EFTs in various limits for the relative masses of M_1, M_2, m^2. For $m^2 \gg M_1$, M_2 we can integrate out the ϕ and ϕ^* fields. Following our leading-order prescription described above, this yields

$$\phi = -\frac{y}{m^2}\bar{\psi}_1\psi_2. \tag{36}$$

Plugging this back into the Lagrangian, we get the effective Lagrangian

$$\mathscr{L}_{eff} = \sum_{i=1}^{2} \bar{\psi}_i(i\gamma^\mu\partial_\mu - M_i)\psi_i + \frac{|y|^2}{m^2}(\bar{\psi}_1\psi_2)(\bar{\psi}_2\psi_1). \tag{37}$$

You'll get to work out the cases $M_1 \gg m^2$, M_2 and $M_1 \sim M_2 \gg m^2$ in the next assignment.

Based on the examples above, it should also be clear that a typical EFT is non-renormalizable. The presence of operators in the Lagrangian with mass dimension greater than four leads to an infinite set of independent divergences. Therefore an infinite set of observables would be needed to renormalize them all. While this might seem to render most EFTs utterly unpredictive, this is not the case. The divergences appearing in an EFT have a very natural physical UV cutoff, namely the large mass scale M suppressing the higher-dimensional operators. As the momentum scale p approaches M, the EFT breaks down anyway (even at tree-level unless an infinite number of terms are included), and it certainly doesn't make sense to work with the EFT at momenta larger than M. From this point of view, the would-be divergences are no longer actually divergent, but they do reflect an unspecified dependence on the underlying UV theory.

One way out of this situation would be to simply work with the full ultraviolet theory, provided it is renormalizable. However, in many cases we only know the low-energy EFT and not the full ultraviolet completion. Even when we don't, working within the EFT is frequently much easier. The trick to handling loops strictly within an EFT is to notice that the new divergences generated correspond to operators of increasingly higher dimension. After regulating all the would-be divergences, we can simply ignore all but a finite set of the operators to any given order in the momentum expansion.

To see how this works in practice, let's first estimate the size of operators for processes with a characteristic momentum p. Ignoring light masses, the only relevant scales are then p and the large UV mass M appearing in higher-dimensional operators. This leads to (at tree-level)

$$(\partial\phi)^2 \sim p^4, \quad m^2\phi^2 \sim m^2p^2, \quad \phi^4 \sim p^4, \quad \frac{\phi^6}{M^2} \sim p^6/M^2 \tag{38}$$

$$\bar{\psi}i\gamma\cdot\partial\psi \sim p^4, \quad m\bar{\psi}\psi \sim m\,p^3, \quad \frac{1}{M^2}(\bar{\psi}\psi)(\bar{\psi}\psi) \sim \frac{p^6}{M^2}. \tag{39}$$

Basically every power of a light field yields a factor of p^n, where n is the mass dimension of that field. Thus, kinetic and dimensionless terms give a baseline operator size of order p^4. Going to lower momentum, we see that mass terms become increasingly important relative to the kinetic terms while higher-dimensional operators become more and more suppressed.

The trick to coming to terms with the non-renormalizability of EFTs is to realize that, in practice, we are only able to measure things to a finite degree of accuracy. Let's say we have a fractional accuracy $\Delta X/X$, for some observable X. If the typical momentum scale for this measurement is p, we can ignore any corrections of size smaller than $(p/M)^N < \Delta X/X$. Thus, in making predictions for X to the required level of accuracy, we can ignore all operators suppressed by powers $n > N$. As a result, we don't care that the non-renormalizability of the theory leads to new "divergences" in operators of dimension greater than N. Only a finite set of operators actually matter when computing to the specified level of accuracy, and we can simply ignore all the rest to an acceptably good level of approximation. Therefore only a finite set of observables are needed to specify the relevant parameters in the theory (based on low-energy data alone), and the theory is predictive for all other observables to the desired level of accuracy.

References

[1] See, for example, R. H. Brandenberger, "Quantum Field Theory Methods and Inflationary Universe Models," Rev. Mod. Phys. **57**, 1 (1985).

[2] M. E. Peskin and D. V. Schroeder, "An Introduction To Quantum Field Theory," *Reading, USA: Addison-Wesley (1995) 842 p*

[3] J. C. Collins, "Renormalization: General theory," [hep-th/0602121]; J. C. Collins, "The Problem of scales: Renormalization and all that," [hep-ph/9510276]; J. C. Collins, "Renormalization. An Introduction To Renormalization, The Renormalization Group, And The Operator Product Expansion," Cambridge, Uk: Univ. Pr. (1984) 380p.

[4] C. P. Burgess and G. D. Moore, "The standard model: A primer," *Cambridge, UK: Cambridge Univ. Pr. (2007) 542 p*

[5] H. Georgi, "Weak Interactions," http://www.people.fas.harvard.edu/ hgeorgi/weak.pdf; H. Georgi, "Review of Effective Field Theory," http://www.people.fas.harvard.edu/ hgeorgi/review.pdf

[6] M. A. Luty, "2004 TASI lectures on supersymmetry breaking," [hep-th/0509029].

[7] D. B. Kaplan, "Five lectures on effective field theory," [nucl-th/0510023].