# P528 Notes #12: Beyond the Standard Model

David Morrissey
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The Standard Model (SM) gives an excellent description of nearly every laboratory experiment performed to date. It provides a quantum mechanical description of all known elementary particles and their interactions with each other (except for gravity). Even so, there are many reasons why the SM cannot be the complete theory of the Universe; there must exist new physics *beyond the Standard Model* (BSM) [1, 2, 3]. In these notes we go over the main motivators for BSM phyiscs, and we outline some of the most promising proposals.

## 1 Gravity

An obvious shortcoming of the SM is that it does not describe the gravitational force. This is almost never a problem in particle physics experiments because gravitational effects are usually completely negligible compared to the other relevant forces. The weakness of gravity can be seen in the size of Newton's constant,

$$G_N \;=\; \frac{1}{8\pi M_{\rm Pl}^2} \;\simeq\; 6.9\times 10^{-39}\ {\rm GeV}^{-2}\ . \tag{1}$$

Here, $M_{\rm Pl} \simeq 2.4\times 10^{18}\,{\rm GeV}$ is the *reduced* Planck mass. The corresponding quantity for the weak force (below about 100 GeV) is the Fermi constant, $G_F = 1.16637\times 10^{-5}\,{\rm GeV}^{-2} = \sqrt{2}g^2/8m_W^2$, which is over 30 orders of magnitude larger. The electromagnetic and strong forces are even stronger than this at low energies.

The SM can be extended to include gravity in a relatively straightforward way [4, 5].[1] Starting from the classical description of gravity, general relativity (GR), the metric field is identified as the dynamical variable and quantum mechanics is applied to it. To do this, one typically starts with a fixed background metric and expands in fluctuations around it.[2] For example, with a flat (Minkowski) background, we would write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/2M_{\rm Pl}\ . \tag{2}$$

A quantum field theory for $h_{\mu\nu}$ can now be built using the standard techniques applied to the action for GR,

$$S = \int d^4x\,\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R(g) + \mathscr{L}_{SM}(g)\right)\ . \tag{3}$$

The quantized excitations of $h_{\mu\nu}$ can be identified with a massless spin-2 graviton interacting with the SM. This theory reproduces GR in this classical limit and agrees with experimental

[1] Sorry for the bad pun.
[2] You may have already done this when studying classical gravitational waves.

data. From now on, when we refer to the SM we will implicitly mean this SM plus quantized-GR theory.

Even though we have a quantum theory of gravity, it is not quite *the* quantum theory of gravity everybody wants. The problem is that this theory is non-renormalizable, with all of the interactions between the graviton and the SM involving higher-dimensional operators suppressed by powers of $M_{\rm Pl}$. As a result, this gravity-extended SM can be trusted as an effective field theory at energies far below $M_{\rm Pl}$, but it loses its predictive power for energies approaching $M_{\rm Pl}$. All current experimental measurements involve single-particle energies well below $M_{\rm Pl}$, so this breakdown of the effective theory has not been a huge problem.

Still, we would very much like to have a quantum theory of gravity that is valid up to energies approaching the Planck scale.[3] This is both a matter of theoretical principle, as well as a necessary step in understanding black holes (which we do have evidence for) and the microscopic structure of spacetime. Discovering the full quantum theory of gravity is still a work in progress, and there are many proposals for what it could be. The best-studied scenario is *superstring theory* in which the elementary constituents can be identified with one-dimensional strings instead of point-like (*i.e.* zero-dimensional) particles [6, 7, 8]. The full implications of string theory are not fully understood, but the theory has been very successful in describing the microscopic structure of black holes [9]. Incorporating the SM within string theory in a consistent way seems to require extra spatial dimensions and supersymmetry. We will discuss aspects of both below. Beyond providing a candidate for QG, work connected to string theory has also led to important developments in quantum field theory, condensed matter physics, and quantum information. Another popular attempt to formulate a quantum theory of gravity is *loop quantum gravity*, in which spacetime emerges in a more dynamical way and without reference to a fixed background metric [10, 11].

A generic expectation for a quantum theory of gravity is that it contains new states with masses near $M_{\rm Pl}$. Some of these heavy states may also have direct couplings to the SM. (String theory exhibits both features.) At energies much lower than $M_{\rm Pl}$, the heavy particles can be integrated out to generate an effective field theory consisting of the SM and graviton fields together with higher-dimensional operators connecting them. The leading terms in the action of the low-energy EFT should match up with those in Eq. (3), and there will also be additional higher-order operators suppressed by powers of $M_{\rm Pl}$. Measuring the effects of such higher-dimensional operators would give us hints about the underlying QG theory.

# 2 The Electroweak Hierarchy Problem

Electroweak symmetry breaking in the SM is induced by an elementary scalar Higgs field $H$. This field develops a vacuum expectation value (VEV) of $\langle H\rangle = v \simeq 174$ GeV that generates masses for the weak vector bosons and the fermions of the theory. The electroweak hierarchy problem is that the VEV of the Higgs field is very sensitive to quantum corrections. Because of this, it is puzzling why the Higgs VEV has the value that it does, rather than being zero

[3]This is what people usually have in mind when they say "quantum gravity" (QG).

Figure 1: Corrections to the scalar (left) and fermion (right) masses at one-loop order.

or much larger.

## 2.1 Scalar Masses are Sensitive

To illustrate the electroweak hierarchy problem, let us return to our simple Yukawa theory containing a real scalar $\phi$ and a massive Dirac fermion $\psi$ connected by a Yukawa coupling,

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \overline{\psi}i\gamma^\mu\partial_\mu\psi - M\overline{\psi}\psi - y\phi\,\overline{\psi}\psi \ . \tag{4}$$

Suppose we want to make the scalar $\phi$ much lighter than the fermion $\psi$, corresponding to $m^2 \ll M^2$. In this limit, let us compute the one-loop correction to the scalar mass squared due to the fermion, corresponding to the first diagram in Fig. 1. It gives

$$\Delta\Gamma^{(\phi^2)}(p) \;\sim\; (\text{divergent}) + \frac{y^2}{(4\pi)^2}M^2\ln\left(\frac{p^2}{M^2}\right) + \ldots + (c.t.) \tag{5}$$

The finite term proportional to $M^2$ is a quantum correction to the squared mass $m^2$ of the scalar. For $M^2 \gg m^2(4\pi/y)^2$, this correction is much larger than the original scalar mass paramter $m^2$ we started with. Now, we can always choose the counterterms to cancel off the large finite correction as well as the formally divergent part, but this result suggests that the *natural* size of the scalar mass $m^2$ is at least as large as $M^2$ times a loop factor. Arranging for $m^2$ to be much smaller than this would appear to require a theoretical *fine-tuning*.

It is also instructive to think about this correction from a low-energy EFT perspective. At low energies $E \ll M$ with $m^2 \ll M^2$, only the scalar degree of freedom is seen directly and we can formulate an EFT containing the scalar alone. To do so, we need to match the effective theory to the full theory at $p^2 \sim M^2$. One of the corrections we should account for is the fermion effect on the scalar mass parameter. This is accommodated by using a different scalar mass in the bare EFT Lagrangian relative to the full theory to make up for the absence of the fermion loop in the EFT. The difference is precisely the finite term in Eq. (5). We call this the *threshold correction* to the scalar mass from integrating out the fermion. Naturalness suggests that the scalar mass-squared parameter in the EFT should not be much smaller than a loop factor times the threshold $M^2$.

The sensitivity of scalar masses to quantum corrections is specific to scalars, and does not arise for fermions or gauge bosons. We can examine the fermion case in our Yukawa theory by taking $m^2 \gg M^2$. The leading quantum correction to the fermion mass mass in this limit comes from the second diagram of Eq. (1), and gives

$$\Delta\Gamma^{(\bar{\psi}\psi)}(p^2) \;\sim\; \text{(divergent)} + \frac{y^2}{(4\pi)^2} M \ln\left(\frac{p^2}{m^2}\right) + \ldots + (c.t.)\ . \tag{6}$$

The situation here is much different from the scalar in that the finite correction to the fermion mass is proportional to the fermion mass itself. This follows from the fact that the theory has a symmetry in the limit $M \to 0$ (or equivalently, $M \to -M$). An important implication of this result is that it is natural for the fermion mass to be arbitrarily small. A similar argument can be applied to gauge bosons, where gauge invariance protects their masses.

## 2.2 The Higgs and the Hierarchy Problem

Going back to the SM, recall that the leading terms in the effective potential for the Higgs field are

$$V = -\mu^2|H|^2 + \frac{\lambda}{2}|H|^4\ . \tag{7}$$

The Higgs VEV corresonds to the value of $|H|$ that minimizes this potential, $v^2 = \mu^2/\lambda \simeq (174\ \text{GeV})^2$. The mass of the physical Higgs boson excitation about this minimum is $m_h = \sqrt{2\lambda\, v^2} = \sqrt{2\mu^2} \simeq 125$ GeV, which has recently been measured experimentally [12, 13]. The electroweak hierarchy problem corresponds to the sensitivity of the $\mu^2$ parameter to quantum corrections.

Suppose we try to extrapolate the SM up to energies much larger than $v$, and let us assume there exists a very heavy new particle $\Psi$ with mass $M_\Psi$ and coupling $y_\Psi$ to the Higgs. Just like in our toy model above, such a particle will induce a finite quantum (loop) correction to the Higgs quadratic parameter in Eq. (7) on the order of [14]

$$\Delta\mu^2 \;\sim\; \mp\frac{y_\Psi^2}{(4\pi)^2} M_\Psi^2\ , \tag{8}$$

where the minus (plus) sign corresponds to $\Psi$ being a fermion (boson).

The electroweak hierarchy problem comes from our expectation that there exist new states $\Psi$ with masses much larger than $\mu$. Such states would imply $\Delta\mu^2 \gg v^2 \sim \mu^2$ as long as $y_\Psi$ is not too small. For example, our attempts at quantum gravity suggest new states with $M_\Psi \sim M_{\text{Pl}}$. If these heavy particles couple directly to the SM (such as can occur in string theory), we would also have $y_\Psi \sim 1$. However, even if the only coupling of the new massive states to the SM is through the massless graviton, we would still expect $y_\Psi^2 \sim (M_\Psi/M_{\text{Pl}})^4/(4\pi)^4$ [14]. In both cases, $\Delta\mu^2$ is much larger than $\mu^2$.

A quantum correction to $\Delta\mu^2$ that is much larger than the observed value of $\mu^2$ is puzzling. To achieve a very small $\mu^2$ relative to $\Delta\mu^2$, the parameters in the underlying high-energy theory must cancel out to a very high precision. From the point of view of the low energy effective theory, there is no good reason why such a cancellation should occur. Thus, our Universe seems to be very finely tuned unless there is new physics that forces $\Delta\mu^2$ to be small. Arguments based on naturalness are therefore a strong motivator for new physics beyond the SM with characteristic mass near the weak scale.

## 2.3 Fix #1: Supersymmetry

Supersymmetry (SUSY) is one of the most popular proposals for new physics beyond the SM. It is an extension of the Poincaré symmetries of flat spacetime that connects particles with different spins. For every known particle, SUSY predicts that there exists a *superpartner* particle with the same charge but with spin differing by half a unit. This connection can provide a solution to the electroweak hierarchy problem. Furthermore, SUSY is an essential component in many attempts to construct a quantum theory of gravity, and can help to explain some of the other shortcomings of the SM to be discussed below.

To illustrate how SUSY addresses the electroweak hierarchy problem, suppose there exists a new fermion $\Psi$ together with its superpartner boson $\widetilde{\Psi}$, both with a coupling $y_\Psi$ to the Higgs field. The equality of this coupling is enforced by supersymmetry. Together, the net leading correction to the Higgs quadratic parameter from this particle-superpartner pair is

$$\Delta\mu^2 \;\simeq\; \frac{y_\Psi^2}{(4\pi)^2}\left(M_{\widetilde{\Psi}}^2 - M_{\widetilde{\Psi}}^2\right) \;. \tag{9}$$

This correction is acceptably small, $\Delta\mu^2 \lesssim \mu^2$, provided the masses of the particle and its superpartner are not too different. Supersymmetry therefore enforces a cancellation of quantum corrections to the Higgs mass parameter. This cancellation can also be thought of as an extension of the chiral protection of fermion masses to their scalar superpartners.

If supersymmetry were an exact symmetry of Nature, every particle would have the same mass as its superpartner, and the leading correction of Eq. (9) would vanish. However, we have not (yet) observed any supersupartners of SM particles, and we certainly would have already if they had the same masses as their SM counterparts. Supersymmetry must therefore be broken, either explicitly or sponatenously. For broken supersymmetry to address the hierachy problem, the breaking must be *soft*, in that all Lagrangian terms that break supersymmetry must be accompanied by a dimensionful parameter $m_{soft}$. This implies that at energies well above $m_{soft}$, the effects of supersymmetry breaking are suppressed by powers of $m_{soft}/p$, and exact supersymmetry becomes an increasingly good approximation.

The soft breaking parameter also determines the typical mass-squared splitting between particles and their superpartners. This implies that the masses of the SM superpartners are on the order of $\sqrt{m_{soft}^2 + m_{SM}^2}$. It also implies that the difference of masses squared in Eq. (9) is approximately $m_{soft}^2$. Taken together, we see that if SUSY is to address the hierarchy problem, we need $m_{soft} \lesssim (4\pi/y_\psi)\mu \lesssim$ TeV. This motivates LHC searches for SM superpartners with masses in the TeV range.

For more detailed discussions of supersymmetry and its applications to particle physics and beyond, I highly recommend the reviews of Refs. [15, 16, 17] and the textbooks of Refs. [18, 19].

## 2.4 Fix #2: New Strong Dynamics

A second approach to the hierarchy problem is to remove the Higgs as fundamental scalar. This can occur if the Higgs scalar is a bound state of fermions (or vector bosons) held together by a new strongly-interacting force. A related possibility, that has been largely ruled out by the discovery of a SM-like Higgs boson, is that the new strong dynamics itself induces electroweak symmetry breaking [20, 21].

Electroweak symmetry breaking through strong dynamics already occurs in the SM through QCD. Consider a simplified version of the SM containing the full $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance, but with no Higgs boson and only a single generation of fermions. In this theory, it might seem that all the fermions are massless and the full $SU(2)_L \times U(1)_Y$ gauge invariance remains manifest. However, it turns out that electroweak symmetry is induced by QCD confinement [20, 21].

To see how this works, recall that the QCD portion of the theory has an approximate $G_{flav} = SU(2)_L \times SU(2)_R \times U(1)_V$ global symmetry, and it is broken down to $SU(2)_V \times U(1)_V$ by QCD confinement at scale $\Lambda_{QCD}$. The electroweak symmetry group $SU(2)_L \times U(1)_Y$ is a gauged subgroup of $G_{flav}$. The gauged and global $SU(2)_L$ parts coincide directly, while hypercharge is generated by

$$Y = t^3_R + \frac{1}{6}\mathbb{I} \ , \tag{10}$$

where the second term coincides with $U(1)_V$ up to an overall normalization. The $U(1)_{em}$ subgroup of the electroweak group is generated by

$$Q = t^3_L + Y = t^3_L + t^3_R + \frac{1}{6}\mathbb{I} \ , \tag{11}$$

as we discussed previously.

As $SU(3)_c$ runs strong at low energies, a quark condensate is generated that spontaneously breaks the approximate global symmetry down to $G_{flav} \to SU(2)_V \times U(1)_V$. In our previous discussion of QCD, this gave rise to three approximate Nambu-Goldstone bosons (NGBs) that we identified with pions. Here, however, we also have electroweak vector bosons present in the theory at the confinement scale. Looking at the gauged subgroup of $G_{flav}$, the quark condensate also breaks $SU(2)_L \times U(1)_Y \to U(1)_{em}$. This suggests three massive vector bosons. I turns out that this occurs through the electroweak vector bosons eating the pions.

To see this explicitly, let us try to write an effective theory for the NGBs. The leading term that is consistent with the global symmetries (in the limit $g$, $g' \to 0$) and the gauged electroweak subgroup is

$$\mathscr{L} \supset \frac{f^2}{4} tr\big(D_\mu \Sigma^\dagger D^\mu \Sigma\big) \ , \tag{12}$$

where

$$D_\mu \Sigma = \partial_\mu \Sigma + igt^a W^a_\mu \Sigma - ig' B_\mu \Sigma\, t^3 \ . \tag{13}$$

This form follows from the embedding of the electroweak group in $G_{flav}$. Note that the $B$ part of the hypercharge component cancels out. Expanding out Eq. (12), the leading terms involving the vector fields are

$$
\begin{aligned}
\mathscr{L} \quad \supset \quad & \frac{f}{2} g W_\mu^+ \partial^\mu \pi^- + \frac{f}{2} g W_\mu^- \partial^\mu \pi^+ + \frac{f}{2} \left( g W_\mu^3 + g' B_\mu \right) \partial^\mu \pi^0 \\
& + \frac{f^2}{4} \left[ g^2 W_\mu^+ W^{\mu -} + (g W_\mu^3 - g' B_\mu)^2 \right] \; .
\end{aligned} \tag{14}
$$

The second line gives the usual mass terms for the weak vector bosons. The bilinear operators in the first line are non-standard, but they signal that that the would-be pions are eaten by the massive vector bosons to generate their longitudinal components. Indeed, we can eliminate these bilinear terms completely by choosing a $SU(2)_L \times U(1)_Y$ gauge such that $\Sigma \to \mathbb{I}$, corresponding to $\Pi^a \to 0$. The resulting $W$ and $Z$ masses are

$$
m_W = \frac{g}{2} f \; , \qquad m_Z = \frac{\sqrt{g^2 + g'^2}}{2} f \; , \tag{15}
$$

which is just like the usual SM expressions but with $v \to f$.

In contrast to electroweak symmetry breaking by a fundamental Higgs field, there is no hierarchy problem for this QCD realization. All the fermions and gauge bosons are effectively massless at high energies by gauge invariance. Going to lower energies, the confinement scale $\Lambda_{QCD}$ is generated by dimensional transmutation. While this solution is elegant, it does not agree with data because it predicts $m_{W,Z} \lesssim 100$ MeV, well below the observed values.

While QCD itself does not work for electroweak symmetry breaking, a scaled up version based on a new non-Abelian gauge group $G_{TC}$ and with new fermions charged under both $G_{TC}$ and $SU(2)_L \times U(1)_Y$ can generate realistic vector boson masses. Such theories are often called *technicolour*, and they require confinement scales of at least few hundred GeV [20], corresponding to $f_{TC} \sim v$. For the most part, these proposed extensions of the SM do not work. They tend to be ruled out by precision electroweak tests and direct searches for the new fermions. They also have trouble generating the observed SM fermion masses, and they usually do no contain a scalar that can be identified with the SM-like Higgs boson seen at the LHC.

A variation on this class of ideas that is still viable relies on the new strong dynamics to generate the Higgs scalar as a composite bound state of more fundamental objects. In such *composite Higgs* scenarios, the resulting Higgs scalar can generate some or all of the fermion masses and electroweak symmetry breaking. Most recent attempts to realize the Higgs boson as a composite state try to identify it with an approximate NGB mode [22]. Recall that NGBs, either exact or approximate, are the exceptions to the expectation of $m \sim f_{TC}$ for bound states. A parametrically lighter pseudo-NGB (pNGB) Higgs boson is attractive for two related reasons. First, it gives a natural separation between the weak scale and $f_{TC}$, and allows us to treat the Higgs field cleanly as a scalar field in an EFT valid below $f_{TC}$. In this EFT, electroweak symmetry breaking is (mostly) induced by the Higgs VEV in much the same way as in the SM. A second consequence of $f_{TC} > m_W$ is that the corrections to precision electroweak observables are not as large. Interestingly, composite Higgs scenarios are closely related to certain theories with an extra spatial dimension [23, 24].

## 2.5 Fix #3: Extra Dimensions

The third approach to the electroweak hierarchy problem is to postulate that the fundamental Planck scale is not much larger than the electroweak scale. In scenarios based on this approach, the strength of gravity is diluted in a way that makes it appear to be much weaker (to us) than it really is, corresponding to a true scale of quantum gravity $M_* \ll M_{\rm Pl}$. While we do not have a full understanding of quantum gravity, it is very likely that our QFT description of elementary particles is unlikely to be valid at energies above $M_*$, and thus there is no hierarchy problem provided $M_* \lesssim 4\pi\, m_W$.

The known mechanisms for diluting the apparent strength of gravity typically make use of extra spacelike dimensions.[4] Such extra dimensions appear to be an essential component of string theories [6, 7, 8], and they have been studied in various other contexts as well [25]. In relation to the hierarchy problem, the two most promising approaches are large extra dimensions, and a warped extra dimension.

In the *Large Extra Dimensions* (LED) scenario, the strength of gravity we see is reduced by a factor of the volume of the extra dimensions [26, 27, 28]. Suppose we have $N$ flat extra dimensions that are periodic with radius $R$, and let $M_*$ be the fundamental Planck scale in the full $d = (4+N)$-dimensional theory. The gravitational potential $\Phi$ in the weak (Newtonian) limit satisfies the Poisson equation

$$\vec{\nabla}^2\Phi \;\sim\; \frac{1}{M_*^{2+N}}\,\rho\;, \tag{16}$$

where $\rho$ is the local energy density. For a pair of static point masses separated by a distance $r$, this leads to a gravitational force of

$$F(r) \;\sim\; \frac{1}{M_*^{2+N}}\frac{m_1 m_2}{r^{2+N}} \qquad (r \ll R)\;, \tag{17}$$

$$\;\sim\; \frac{1}{M_*^{2+N}}\frac{m_1 m_2}{r^2(2\pi R)^N} \qquad (r \gg R)\;. \tag{18}$$

For $r \ll R$, the gravitational flux can now spread out in more ways than $d = 4$ leading to a faster decrease of the force with distance. However, for $r > R$ the extent to which the flux lines can spread is limited by the size of the extra dimensions, and the familiar $1/r^2$ behaviour of $d = 4$ is regained. Matching the long-distance expression to Newton's force law, we can also identify

$$M_{\rm Pl}^2 = (2\pi R)^n M_*^{2+N} = V_N M_*^{2+N}\;, \tag{19}$$

where $V_N$ is the total volume of the compact extra dimensions.

The idea of Refs. [26, 27, 28] was to use this dilution to recast the hierarchy problem by making $R$ large enough that $M_* \sim$ TeV.[5] For $n$ extra dimensions of equal size, the required

[4] Timelike extra dimensions lead to challenges with causality and such.

[5] To fully solve the hierarchy problem, a mechanism to fix the large radii is also needed [29].

radius $R$ is

$$2\pi R \;\simeq\; 10^{32/n}10^{-17}\text{cm} \;\sim\; \left\{ \begin{array}{ccccc} 10^{15}\,\text{cm} & (R^{-1}\sim ...) & ; & N=1 \\ 1\,\text{mm} & (R^{-1}\sim 10^{-13}\text{ GeV}) & ; & N=2 \\ 1\,\mu\text{m} & (R^{-1}\sim 10^{-8}\text{ GeV}) & ; & N=3 \\ 10\,\text{fm} & (R^{-1}\sim 10^{-2}\text{ GeV}) & ; & N=6 \end{array} \right. . \tag{20}$$

These radii are very large compared to typical particle physics scales. For this reason, scenarios of this type are referred to as *large extra dimensions* (LED) or ADD after the original authors [26, 27, 28].

With a *Warped Extra Dimenson*, gravity appears to be extremely weak because it is localized away from us in the extra dimension [30, 31]. The standard example is the Randall-Sundrum (RS) model, in which there is a single extra dimension of finite size $w \in [0, \pi\, r_c]$ with a net spacetime curvature described by the metric [30, 31]

$$ds^2 = G_{MN}dx^M dx^N = e^{-2kw}\eta_{\mu\nu}dx^\mu dx^\nu - dw^2 \;, \tag{21}$$

where $k$ characterizes the curvature. This spacetime is illustrated in Fig. 2, and is said to be *warped*, with the boundary at $w = 0$ called the *UV brane*, the boundary at $w = \pi r_c$ called the *IR brane*, and the space in between called the *bulk*.

The RS spacetime is a solution of Einstein's equations in $d = 5$ with a bulk cosmological constant [30, 31]. At long distances, $r \gg r_c$, the gravitational force law between a pair of point masses in this spacetime reduces to the standard Newtonian form with an effective (reduced) Planck scale of

$$M_{\text{Pl}}^2 = \frac{M_*^3}{k}\left(1 - e^{-2\pi k r_c}\right) \;. \tag{22}$$

For moderate values of $kr_c$ and $M_* \sim k$, this gives an effective $d = 4$ Planck scale of about the same order as the $d = 5$ Planck scale $M_*$ and the curvature $k$. Thus, there is no strong volume dilution as in LED.

A warped extra dimension can address the electroweak hierarchy problem if the Higgs field is localized on the IR brane at $w = \pi r_c$. To see how, consider the action for the Higgs field (generalized to be consistent with general relativity):

$$\begin{aligned} S_{Higgs} &= \int d^4x \int_0^{\pi r_c} dw\, \sqrt{G}\left[G^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - V(|H|)\right]\,\delta(w-\pi r_c) && (23)\\ &= \int d^4x\; e^{-4\pi k r_c}\left[e^{2\pi k r_c}\eta^{\mu\nu}\,\partial_\mu H^\dagger \partial_\nu H - V(|H|)\right] && (24)\\ &= \int d^4x\;\left[\eta^{\mu\nu}\,\partial_\mu \widetilde{H}^\dagger \partial_\nu \widetilde{H} - e^{-4\pi k r_c}V(|e^{\pi k r_c}\widetilde{H}|)\right] \;, && (25) \end{aligned}$$

where in the last line we have changed variables to $H = e^{\pi k r_c}\widetilde{H}$ to make the kinetic term canonical. If the original Higgs potential for $H$ takes the standard form, $V(|H|) = -\mu^2|H|^2 + \lambda|H|^4/2$, the rescaled potential in terms of the new variables is

$$e^{-4\pi k r_c}V(|e^{\pi k r_c}H|) \;\;=\;\; -\left(\mu^2 e^{-2\pi k r_c}\right)|\widetilde{H}|^2 + \frac{\lambda}{2}|\widetilde{H}|^4 \;. \tag{26}$$

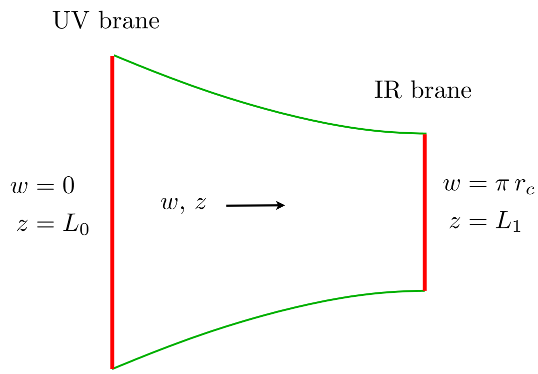


Figure 2: Simple picture of the RS spacetime.

The key feature here is that the dimensionful Higgs mass parameter $\mu^2$ has been warped by the factor $e^{-2\pi k r_c}$. Naturalness suggests $\mu^2 \sim M_*^2 \sim M_{\rm Pl}^2$. Thanks to the warp factor, the natural value of the effective Higgs mass parameter on the IR brane can be near the weak scale provided the warp factor is big enough, which corresponds numerically to $kr_c \sim 11$:

$$\mu_{eff} \ \sim \ M_* e^{-\pi k r_c} \ \sim \ \text{TeV} \qquad (kr_c \sim 11) \ . \tag{27}$$

More generally, any dimensionful quantity localized at $w$ in the bulk gets rescaled by $e^{-\pi k w}$. This rescaling is closely related to scaling of operators by renormalization group effects in conformal field theories [23, 24], as you studied in question #3 of `hw-08`.

# 3 Cosmology

If the SM-plus-gravity quantum effective theory of Eq. (3) is the complete theory of elementary particles at energies below $M_{\rm Pl}$, it should be able to account for the large-scale structure of the Universe. However, detailed cosmological measurements do not agree with the predictions of the SM. There are three main puzzles: the extreme flatness and spatial uniformity of the Universe, the apparent need for dark matter, and the excess of matter over antimatter. All three are strongly suggestive of new physics below the Planck scale.

The Universe is extremely uniform over very large distances. This is seen best in the cosmic microwave background (CMB) radiation, which consists of photons with a mean effective temperature of about $T \simeq 2.73\,\text{K} \simeq 2.4 \times 10^{-13}$ GeV [32]. Relative to the mean value, the primordial fluctuations in the CMB temperature are very small – only about one part in $10^5$. Extrapolating the CMB back in time using the SM and GR implies that the early Universe consisted of a very hot plasma of subatomic particles. The energy density of this plasma caused the Universe to expand, which in turn caused the plasma to cool. When the plasma temperature fell below $T \simeq 0.3\,\text{eV}$, its charged components (mostly protons and electrons) combined to form neutral atoms. This quickly depleted the electrically-charged fraction of the plasma, a process called *recombination*, and allowed the remaining photons to travel across the Universe unimpeded. What we see today as the CMB are these leftover photons, and therefore the CMB gives us a snapshot of the Universe when it was much younger.

The uniformity of the CMB is curious because the extrapolation of the SM back in time also implies that many different regions of the CMB we see today were causally disconnected when the CMB was formed at recombination. From this point of view, it is very surprising that these regions should be so close in temperature. The leading resolution of this puzzle is *inflation*, a period of rapid exponential expansion of spacetime in the very early Universe [33]. Inflation allows a small causally connected patch of spacetime to be expanded so much that it makes up the entire visible sky today. Most theories of inflation introduce a new scalar field to the SM with a very flat potential [33].[6] If this *inflaton* scalar field starts off with a large displacement from the minimum of its potential, the potential energy of the field can drive a period of inflation. The expansion from inflation dilutes away everything that was present in the Universe before it began. Eventually, the inflaton field rolls down to the minimum of the potential, oscillates for a bit, and transfers its energy back to radiation when it decays in a process called *reheating*. The end result of inflation and reheating is a very hot and uniform thermal plasma of subatomic particles; this is precisely what we observe.

Inflation makes other predictions that agree with cosmological observations. Measurements of the CMB show that the net spatial curvature of the Universe is zero to within a small experimental uncertainty. This is also expected from inflation, since any initial curvature would be strongly diluted by the exponential inflationary expansion. The spectrum of small temperature fluctuations in the CMB is consistent with and expected from inflation. In particular, they can be understood as coming from quantum fluctuations in the inflaton field during inflation. While these various measurements agree with the general predictions of inflation, we still do not have enough information to deduce the underlying theory of the inflaton.

Detailed studies of the CMB together with other cosmological and astrophysical observations point towards additional shortcomings of the SM. One of the most striking is that the total density of matter (*i.e.* non-relativistic particles) appears to be much larger than what can be accounted for by the SM [35, 36, 37]. The evidence for this extra *dark matter* comes from a diverse set of observations over a very broad range of distances. These include galactic rotation curves, the average motion of galaxy clusters, the structure of the distribution of matter, and the fluctuations in the CMB.

The most simple explanation for dark matter is the existence of a new massive particle with a moderate cosmological density. To account for observations, it must be electrically neutral and uncharged under the strong force, but other than that we know very little about what such a particle could be. Many theories of BSM physics predict or can accommodate a dark matter candidate. In particular, the observed DM density can plausibly be explained by a weakly-interacting massive particle (WIMP), a new particle with mass close to the weak scale and weak interactions with the SM [35, 36, 37]. Such WIMPs arise frequently

[6] The SM Higgs may also induce inflation if it has a non-minimal coupling to gravity [34].

in BSM theories that address the electroweak hierarchy problem. On the experimental side, searches for dark matter are underway using a wide range of techniques including particle colliders, deep underground direct dark matter detectors, and observations of cosmic rays in our galaxy [37]. None of these experiments has found anything conclusive yet.

The observed density of SM matter in the Universe is also puzzling. By energy, it is dominated by baryons (in the form of protons and light nuclei). and it consists almost entirely of matter rather than antimatter [38]. The origin of this *baryon asymmetry* is a mystery, and there is no known way to generate it within the SM alone. In contrast, there are a number of viable mechanisms to generate the baryon asymmetry in BSM theories [38].

A fourth puzzle related to cosmology is the *cosmological constant problem* [39]. Most of the energy density in the Universe today (about 75%) seems to come from a positive net value of the background vacuum energy. This vacuum energy can be accommodated within the SM by adding a constant term to Eq. (3),

$$S \;\to\; S - \int d^4x\sqrt{-g}\;\Lambda_{cc}\;, \tag{28}$$

where $\Lambda_{cc}$ is called the *cosmological constant* (CC). Note that without gravity, the CC would not have any physical effects. However, in GR it acts as a source for spacetime curvature and must be taken into account. The problem with the CC is its size (determined from observation), $\Lambda_{cc} \simeq (2.5\times10^{-12}\ \text{GeV})^4$. This is absolutely miniscule compared to the natural value of $M_{\text{Pl}}^4$, or any other dimensionful scale within the SM for that matter. It is not clear how to explain this vast difference.

# 4 Flavour and CP Violation

The SM has three generations of quarks and leptons with a very wide range of masses. Weak interactions induce a mixing between these different *flavours* (or *generations*) of fermions, and they allow the heavier flavours to decay to the lighter ones and mediate CP violation. While the SM is able to accomomdate this structure, it does not explain it.

The range of fermion masses in the SM is enormous, from sub-eV for the neutrinos, to $m_e \simeq 0.511 \times 10^{-3}$ GeV for the electron, and up to $m_t \simeq 174$ GeV for the top quark. Quark mixing via the weak interactions is described by the unitary Cabbibo-Kobayashi-Maskawa (CKM) matrix, whose numerical values are [40],

$$|V_{CKM}| \;=\; \begin{pmatrix} 0.9743(2) & 0.2253(8) & 0.0041(5)\\ 0.225(8) & 0.99(2) & 0.041(1)\\ 0.008(1) & 0.040(3) & 1.02(3)\end{pmatrix}\;. \tag{29}$$

These entries are seen as being very suggestive of an underlying hierarchical structure. The CKM matrix also contains phases that give rise to CP violation.

All the charged fermion masses and mixings in the SM come from Yukawa couplings to the Higgs of the form

$$-\mathscr{L} \supset y^f_{ij} \overline{F}^i_L f^j_R \overset{(\sim)}{H} + (h.c.) \ , \tag{30}$$

where $F_L$ is a $SU(2)_L$ doublet, $f_R$ is a right-handed $SU(2)_L$ singlet, $H$ is the Higgs, and $i$, $j$ are flavour indices that run over the three generations. From this perspective, a theory for the underlying structure of flavour and CP is equivalent to theory for the origin of the Yukawa couplings $y^f_{ij}$.

The multi-generational structure of the SM allows for CP violation. Observations of CP violation in quark-mediated processes is consistent with the possible phases of the CKM matrix. These phases emerge from complex values of the Yukawa couplings in Eq. (30), and at least three generations of quarks are needed for them to produce observable effects. For this reason, it is reasonable to expect that an explanation for the origin CP violation might be related to the origin of quark flavour.

The SM has another source of CP violation beyond the CKM matrix. It is the *QCD* $\Theta$ *parameter*, corresponding to the coefficient of the operator [41]

$$-\mathscr{L} \ \supset \ \frac{\Theta}{32\pi^2}\, \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \ , \tag{31}$$

where $G^a_{\mu\nu}$ is the gluon field strength. If all the quark masses are non-zero (which appears to be the case experimentally), the $\Theta$ parameter can give rise to observable CP-violating effects. In particular, it can induce an electric dipole moment (EDM) for the neutron. Attempts to measure such an EDM have found nothing so far implying $|\Theta| \lesssim 10^{-10}$ [41]. The required smallness of this parameter has no good explanation in the SM, and is called the *strong CP problem*. The most promising solution is to introduce a new pseudoscalar *axion* particle that dynamically drives $\Theta \to 0$ [42].

# 5 Neutrino Masses and Mixings

Neutrinos (and antineutrinos) in the SM are predicted to be massless and to have one of three definite flavours corresponding to the $e$, $\mu$, and $\tau$ charged leptons. As discussed previously in the course, there is no leptonic equivalent of the CKM quark mixing matrix, and lepton flavour is conserved in the SM. However, detailed measurements of neutrinos have detected the phenomenon of *neutrino oscillations*, in which a neutrino of one type transforms into another. These oscillations are definitive proof of new physics beyond the SM, and they imply further that at least some of the SM neutrinos have mass [43, 44, 45].

In contrast to the other SM fermions, the neutrinos interact exclusively through the weak vector bosons. This makes them much harder to detect than the other SM fermions, and allows them to travel very long distances through matter without being scattered. The flavour of a neutrino when it is produced is deduced from the flavour of the charged lepton that is created (or decayed) along with it. Similarly, neutrinos are "detected" when they scatter with other matter, often in conjunction with a charged lepton. Neutrino oscillations are observed in a difference between the neutrino flavour at detection relative to production.

To illustrate how neutrino oscillations work, consider a simple two-state quantum mechanics system with *flavour* eigenstates $|\nu_e\rangle$ and $|\nu_\mu\rangle$ and *mass* eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$. The flavour states are those produced in conjunction with the corresponding charged leptons, while the mass states are energy eigenstates with energies equal to $m_i$ in the particle rest frame. Neutrino oscillations can occur when the flavour and mass eigenstates do not line up,

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} . \tag{32}$$

Suppose a $|\nu_e\rangle$ state is created at $(t, \vec{x}) = (0, \vec{0})$ with a 4-momentum $p$ that is much larger than $m_1$ and $m_2$. At later times, it will evolve into

$$\begin{aligned} |\nu_e(x)\rangle &= e^{-ip\cdot x}\,|\nu_e\rangle \\ &= e^{-ip_1\cdot x}\,|\nu_1\rangle + e^{-ip_2\cdot x}\,|\nu_2\rangle . \end{aligned} \tag{33}$$

The probability to detect the $|\nu_e\rangle$ state at a detector a distance $L$ from the production point is then

$$\begin{aligned} |\langle\nu_e|\nu_e(L)\rangle|^2 &= 1 - \sin^2(2\theta)\ \sin^2\!\left(\frac{\Delta m^2 L}{4E}\right) \\ &= 1 - \sin^2(2\theta)\ \sin^2\left[1.27\left(\frac{\Delta m^2}{\mathrm{eV}^2}\right)\left(\frac{L}{\mathrm{km}}\right)\left(\frac{\mathrm{GeV}}{E}\right)\right] , \end{aligned} \tag{34}$$

where $\Delta m^2 = m_2^2 - m_1^2$. To derive this result, the standard approximation is to use $p_i \cdot x \simeq (E_i - p_i)L \simeq m_i^2 L/2E$ [43]. This is a bit of cheat, but Eq. (34) is the correct result of a proper derivation using density matrices and wave packets [44, 45].

The result of Eq. (34) illustrates the necessary ingredients for neutrino oscillations. Specifically, the neutrinos must have different masses ($\Delta m^2 \neq 0$), and the mass and flavour eigenstates must be misaligned $\theta \neq 0$. When these conditions are met, the probability to observe the initial $|\nu_e\rangle$ neutrino flavour in the detector varies with the neutrino energy and the detector distance. There is also a non-zero *appearance* probability to detect the second $|\nu_\mu\rangle$ flavour, which by unitarity is

$$P_{e\mu} = 1 - P_{ee} = \sin^2(2\theta)\ \sin^2\!\left(\frac{\Delta m^2 L}{4E}\right) . \tag{35}$$

Some experiments measure appearance, others disappearance, and a few do both.

Oscillations among all three flavours of SM neutrinos have been observed. The generalization of the two-flavour mixing described above to the full system is given by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix,

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} , \tag{36}$$

with the unitary mixing matrix $U_{PMNS}$ typically decomposed according to

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{37}$$

This decomposition is useful because the various mixing angles have been measured in different systems [40]. The $\theta_{12} \simeq 35°$ angle is determined best from measurements of neutrinos emitted by nuclear reactions in the sun, and is sometimes called the solar mixing angle. Neutrinos obtained from cosmic ray showers in the atmosphere gave the first good determination of $\theta_{23} \simeq 45°$, and it is sometimes called the atmospheric mixing angle. Recent measurments of neutrinos produced in nuclear reactors have yielded $\theta_{13} \simeq 14°$. Oscillation measurements also give values for the mass differences of neutrinos, with

$$\Delta m_{12}^2 \simeq 7.6 \times 10^{-5}\,\mathrm{eV}^2 \ , \qquad |\Delta m_{23}^2| \simeq 2.4 \times 10^{-3}\,\mathrm{eV}^2 \ . \tag{38}$$

There is also a limit on the sum of the SM neutrino masses from cosmological observations of $\sum_i m_i \lesssim 0.2\,\mathrm{eV}$ [46]. Compared to the other fermions of the SM, the neutrinos are orders of magnitude lighter, and their mixings are significantly larger than those of the CKM matrix.

Neutrino masses and mixings require new physics beyond the SM (as we have defined it). The easiest way to generate them is to add three gauge single right-handed neutrinos $N_R = (1, 1, 0)$ with the Yukawa coupling

$$-\mathscr{L} \ \supset \ y_{N_{AB}} \overline{L}_{L_A} \tilde{H} N_{R_B} + (h.c.) \ . \tag{39}$$

After electroweak symmetry breaking, $\tilde{H} \to (0, v + h/\sqrt{2})$, this generates a neutrino mass matrix with entries

$$(m_\nu)_{AB} = (y_N\, v)_{AB} \ . \tag{40}$$

In the end, we get three massive Dirac neutrinos and a mixing matrix connecting them to the charged leptons via the $W$ boson. However, given the extreme smallness of the observed neutrino masses, many consider this solution unsatisfying on its own.

A popular variation on the simple picture above is called the (Type-I) neutrino seesaw. Since the $N_R$ are gauge singlets, we can also add Majorana masses for them of the form

$$-\mathscr{L} \ \supset \ \frac{1}{2} M_{N_{AB}}\, \overline{(N_{R_A}^c)} N_{R_B} + (h.c.) \ . \tag{41}$$

Combined with the Dirac mass term of Eq. (39), the full neutrino mass matrix takes the schematic form

$$M_\nu = \begin{pmatrix} 0 & y_N v \\ y_N v & M_N \end{pmatrix} \ . \tag{42}$$

For $M_N \gg y_N v$, the mass eigenstates then consist of six Majorana fermions with mass eigenvalues of the form

$$m_\nu \simeq \ \frac{(y_N v)^2}{M_N} \ , \quad M_N \ . \tag{43}$$

The three light states are identified with the SM neutrinos, while the three heavy neutrinos are mostly singlets and very difficult to detect. For $y_N \sim 1$, the SM-like neutrinos have sub-eV masses for $M_N \sim 10^{13}$ GeV.

It is also interesting to look at the EFT obtained by integrating out very massive right-handed neutrinos. The leading operator generated from doing so is

$$-\mathscr{L}_{EFT} \supset \frac{1}{2M_N}\overline{(L^c_{LA})} y_{N_{AB}} y_{N_{BC}} L_{LC} + (h.c.)\ . \tag{44}$$

This is the lowest-dimensional non-renormalizable operator that can be built out of SM fields alone. After electroweak symmetry breaking, it generates neutrino masses on the order of $m_\nu \sim y_N^2 v^2/M_N$, as expected from the neutrino seesaw.

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