PHYS 528 Homework #9

Due: Apr.8, 2021

1. Read `notes-10`, `notes-11`, and `notes-12`, posted on the course webpage. Did you read them? (Y/N – almost free points!)

2. The identification of current operators with pion fields implies an effective pion interaction with leptons (via the weak force) of

$$-\mathscr{L}_{eff} \supset 2 f_\pi G_F \, \partial_\mu \pi^- \left(\bar{\ell}\gamma^\mu P_L \nu_\ell\right) + (h.c.) \, ,$$

   where ℓ is any of the SM charged lepton fields. This interaction corresponds to the vertex $-ip_\pi^\mu \gamma_\mu P_L$. Use this to compute the decay widths for $\pi^- \to \mu\bar{\nu}_\mu$ and $\pi^- \to e\bar{\nu}_e$. Put in numbers and compare your results to the PDG (http://pdglive.lbl.gov/).
   Hint: $f_\pi \simeq 93\,\text{MeV}$ *and* $G_F \simeq 1.17 \times 10^{-5}\ \text{GeV}^{-2}$.

3. Chiral perturbation theory (with two massless quarks).

   a) Work out the transformation properties of the Π fields to linear order in c_V^a and c_A^a for the two cases $c_A^a = 0$ ($SU(2)_V$) and $c_V^a = 0$ (broken generators).

   b) Expand the leading ($\mathcal{O}(p^2)$) term in the chiral perturbation theory Lagrangian to quadratic order in the Π fields and verify explicitly that you get canonical kinetic terms for the pions.
   Hint: $\sigma^a\sigma^b = \delta^{ab}\mathbb{I} + i\epsilon^{abc}\sigma^c$, $tr(\sigma^a\sigma^b) = 2\delta^{ab}$.

   c) Find the Noether currents for $SU(2)_L$, $SU(2)_R$, and $U(1)_V$ in the low-energy EFT. Expand these to leading non-trivial order in the Π fields.
   Hint: treat Σ *and* $\Sigma^\dagger$ *as the dynamical fields.*

   d) Expand out to leading order the term

$$-\mathscr{L} \supset \frac{1}{2}\tilde{\Lambda}^3 tr(M\Sigma) + h.c \, ,$$

   where $M = diag(m_u, m_d)$, and show that it produces mass terms for the pions.
   Hint: recall that $\pi^- = (\pi^+)^*$ *is complex, and that the mass term for a complex scalar does not have a* $1/2$ *factor.*

4. Complete the course evaluation at https://particletheory.triumf.ca/PHYS528/TRAC.pdf and email it to trac@triumf.ca. (Optional, but I won't see it. UBC students will also have a UBC evaluation.)