PHYS 528 Homework #8

Due: Mar.22, 2024, 12pm PDT

1. Higgs decays to fermions.

   a) Use Eq. (34) of `notes-06` to deduce the vertex corresponding to the Higgs coupling to bottom quarks. Don't forget quark colours!

   b) Apply this to compute the decay rate of the Higgs boson to $b\bar{b}$.

   c) Compare this result to the decay rates for Higgs to $c\bar{c}$ and $\tau\bar{\tau}$. To do so, find the numerical values of these widths (in MeV) for a Higgs mass of $m_h = 125$ GeV.

   Hint: you can look up fermion masses at https://pdglive.lbl.gov/.
   Hint: don't forget to take colours into account.

2. Decays of the top quark.

   a) The top quark decays nearly all the time via $t \to W^+ + b$. Based on the size of the coupling of the top to the W^+ in the SM Lagrangian, make an educated guess for the decay width and explain the reasoning behind it. To makes things easier, for the estimate assume that the $m_t \gg m_W$, m_b so that the only relevant energy scale is the top mass.
   Note: do this before parts b) and c); no cheating!

   b) Compute the decay width explicitly, and compare it to the measured value. You may treat the b quark as being massless to make your life easier.

   c) What happens to the top decay width as m_t becomes much larger than m_W? Compare this to the rough estimate in part a).

3. The narrow width approximation.

   Consider a theory consiting of a massless "electron," a fermion f of mass m, and a real scalar of mass $M > 2m$, with interactions

   $$-\mathscr{L} \supset g_e\,\phi\,\bar{e}e + g_f\,\phi\,\bar{f}f\ .$$

   In this theory:

   a) Compute the decay widths of ϕ into $e\bar{e}$ and $f\bar{f}$, and the total decay width Γ.

   b) Calculate the cross section for $e\bar{e} \to \phi$ in the CM frame. You should get a leftover delta function. Rewrite it as a delta function on the variable $s = (p_1+p_2)^2$, where p_1 and p_2 are the initial momenta, and express the rest of the cross section in terms of of the decay width $\Gamma(\phi \to e\bar{e})$ and the mass M.

   c) Find the total cross section for $e\bar{e} \to f\bar{f}$ in the CM frame. In doing so, use the width-corrrected propagator

   $$\text{Prop} = \frac{i}{p^2 - M^2 + iM\Gamma}\ .$$

d) For small couplings g_e and g_f, we will have $\Gamma \ll M$. In this limit, we can apply the *narrow width approximation*,

$$\lim_{\Gamma/M\to 0} \frac{1}{(s-M^2)^2+M^2\Gamma^2} = \frac{\pi}{M\Gamma}\delta(s-M^2)\ .$$

Use this approximation to rewrite the $e\bar{e} \to f\bar{f}$ cross section in terms of $\sigma(e\bar{e} \to \phi)$ and $\text{BR}(\phi \to f\bar{f}) = \Gamma(\phi \to f\bar{f})/\Gamma$.

e) **(Optional - will not be marked!)**
These calculations can be generalized to any theory with a narrow s-channel resonance. Use this to argue that measuring the cross section for $e\bar{e} \to f\bar{f}$ (with $f \neq e$) in the CM frame with $s = m_Z^2$ lets us extract $\Gamma(Z \to f\bar{f})$.
Note: you don't need to give a proof but you should make a plausible argument.
Note: feel free to make the argument for the specific case of a scalar resonance only, but note that it can be generalized to higher spins as well.

4. $e^+e^- \to f\bar{f}$ scattering, $f \neq e$ **(Optional – will not be marked!)**

a) Find the amplitude for this process for a general SM fermion f (that is not the electron e) from the diagram with the photon in the s-channel. Use this to compute the part of summed and squared amplitude from the photon contribution, "$|\mathcal{M}_\gamma|^2$". Here and for the rest of this question, you may neglect fermion masses. Also, show your calculations.

b) Find the amplitude for this process from the diagram with the Z^0 in the s-channel. Use this to compute "$|\mathcal{M}_Z|^2$" from the Z^0 contribution.
Hint: there are lots of terms, but many of them vanish!
Hint: $P_L P_R = 0$, $P_L^2 = P_L$, $P_L\gamma^\mu = \gamma^\mu P_R$, $tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) = -4i\epsilon^{\mu\nu\rho\sigma}$.
Hint: $\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\rho\sigma} = -2(\delta^\mu{}_\rho\delta^\nu{}_\sigma - \delta^\mu{}_\sigma\delta^\nu{}_\rho)$

c) Compute the summed and squared interference term "$[\mathcal{M}_\gamma\mathcal{M}_Z^* + (h.c.)]$".

d) Combine these to find $d\sigma/d\cos\theta$ in the CM frame, where θ is the angle of the outgoing fermion relative to the direction of the incoming electron.

e) Which contribution dominates for $s = (p_1+p_2)^2 \ll m_Z^2$? Which dominates for $s \simeq m_Z^2$? What is the asymptotic behaviour for $s \gg m_Z^2$?