PHYS 528 Homework #2

Due: Feb.2, 2024, 12pm PST

1. Work out the differential cross section $d\sigma/d(\cos\theta)$ for the process $e^+e^- \to \mu^+\mu^-$, where θ is the CM-frame angle between the incident electron and the outgoing muon. You may work in the limit that $p^2 \gg m_e^2$ and ignore the electron mass ($m_e \to 0$), but do keep the full dependence on the muon mass (instead of dropping it like we did in class). *Hint: note that the differential cross is what you get from the cross section formula but without doing the final integral over the outgoing angle θ.*

2. Compute the summed and squared matrix element for $e^-\mu^- \to e^-\mu^-$ scattering to leading order in QED at very high energy, $E_{CM} \gg m_\mu,\, m_e$. This implies that you can neglect the fermion masses.
(**Optional: use this to compute the differential and total cross sections.**)
Hint: recall that the photon couples diagonally to lepton flavour in the sense that there are γee and $\gamma\mu\mu$ vertices but no $\gamma\mu e$ vertices.

3. Consider a massive Z' vector boson that couples to electrons with a vertex factor equal to $-ig'\gamma^\mu$ for some dimensionless coupling g'.

   a) A massive vector has three independent polarization states. These can represented by any three independent unit 4-vectors $\epsilon_\mu(p,\lambda)$ satisfying the constraints $p^\mu\epsilon_\mu = 0$ and $\epsilon^*_\mu(p,\lambda)\epsilon^\mu(p,\lambda') = -\delta_{\lambda\lambda'}$, where p^μ is the four-momentum of the vector boson and $\lambda = 1, 2, 3$ labels the three different polarizations. Find a simple set of polarization vectors in the rest frame of the massive vector. Show that they satisfy the constrained completeness relation

   $$\sum_\lambda \epsilon_\mu(p,\lambda)\epsilon^*_\nu(p,\lambda) = -\eta_{\mu\nu} + p_\mu p_\nu/m_{Z'}^2.$$

   Hint: choose the z-axis to align with the directions of the outgoing electrons.

   b) Compute the total unpolarized decay width for $Z' \to e^+e^-$. For this, use the vertex and the completeness relation stated above. Keep the full dependence on the masses of the Z' and the electron.