PHYS 528 Homework #7

Due: Mar.23, 2021

1. Decays of the top quark.

   a) The top quark decays nearly all the time via $t \to W^+ + b$. Based on the size of the coupling of the top to the W^+, make an educated guess for the decay width and explain the reasoning behind it.

   b) Compute the decay width explicitly, and compare it to the measured value. You may treat the b quark as being massless to make your life easier.

   c) What happens to the top decay width as m_t becomes much larger than m_W? Compare this to the rough estimate in part a).

2. The narrow width approximation.

   Consider a theory consiting of a massless "electron," a fermion f of mass m, and a real scalar of mass $M > 2m$, with interactions

   $$-\mathscr{L} \supset g_e\,\phi\,\bar{e}e + g_f\,\phi\,\bar{f}f\ .$$

   In this theory:

   a) Compute the decay widths of ϕ into $e\bar{e}$ and $f\bar{f}$, and the total decay width Γ.

   b) Calculate the cross section for $e\bar{e} \to \phi$ in the CM frame. You should get a leftover delta function. Rewrite it as a delta function on the variable $s = (p_1+p_2)^2$, where p_1 and p_2 are the initial momenta, and express the rest of the cross section in terms of of the decay width $\Gamma(\phi \to e\bar{e})$ and the mass M.

   c) Find the total cross section for $e\bar{e} \to f\bar{f}$ in the CM frame. In doing so, use the width-corrrected propagator

   $$\text{Prop} = \frac{i}{p^2 - M^2 + iM\Gamma}\ .$$

   d) For small couplings g_e and g_f, we will have $\Gamma \ll M$. In this limit, we can apply the *narrow width approximation*,

   $$\lim_{\Gamma/M\to 0}\ \frac{1}{(s-M^2)^2 + M^2\Gamma^2} = \frac{\pi}{M\Gamma}\delta(s-M^2)\ .$$

   Use this approximation to rewrite the $e\bar{e} \to f\bar{f}$ cross section in terms of $\sigma(e\bar{e} \to \phi)$ and $\text{BR}(\phi \to f\bar{f}) = \Gamma(\phi \to f\bar{f})/\Gamma$.

3. Z-pole asymmetries.

   a) Prove the expression for the left-right asymmetries in terms of the effective Z couplings given in `notes-07` (*i.e.* Eq. (16) starting from Eq. (15) and applying Eq. (9)).

   b) Compute the numerical values of the left-right asymmetries A_f ($f \neq e$) and compare to data.
   Hint: look up "Asymmetry Parameters" under the Z listing here: pdglive.lbl.gov/.

   c) Prove the relation between the forward-backward asymmetries and the left-right asymmetries at the Z pole discussed in class. At the pole, you may neglect the photon contribution.