SLAC NATIONAL ACCELERATOR LABORATORY

Axion Dark Matter Detection in an RF Cavity

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Based on:
1912.11048
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Outline

Axion couplings to photons

Existing detection strategy overview

Radio-Frequency **up-conversion** approach

Signal

Noise:

Standard noise sources

Non-standard noise sources

Outlook

Axion couplings to photons

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F \tilde{F}$$

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QCD axion inevitably has such a coupling

$$g_{a\gamma\gamma}^{\mathrm{QCD}} \simeq \frac{\alpha}{2\pi} \frac{1}{f_a} \left(\frac{E}{N} - 1.92 \right)$$

DFSZ: $\frac{E}{N} = \frac{8}{3}$

KSVZ: $\frac{E}{N} = \begin{cases} 0 & \text{neutral VLQs} \\ 2 & \pm 1 \text{ charged VLQs} \end{cases}$

Anomaly coefficients

Mixing w/ pion

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ALP has coupling to photons introduced "by hand"

$$g_{a\gamma\gamma}^{\text{ALP}} \simeq \frac{\alpha}{2\pi f_a}$$

Resonant Axion Searches

Axion electrodynamics: $\mathcal{L} \supset -\frac{g_{a\gamma\gamma}}{4}\, a\, F\tilde{F} = -g_{a\gamma\gamma}\, a\, \mathbf{E}\cdot\mathbf{B}$

$$\nabla \cdot \mathbf{E} = \rho - g_{a\gamma\gamma}\mathbf{B}\cdot\nabla a$$

$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J} - g_{a\gamma\gamma}\left(\mathbf{E}\times\nabla a - \mathbf{B}\partial_t a\right)$$

Maxwell's new and improved Equations

Axion dark matter: $a(t) \simeq \frac{\sqrt{2\rho_{\rm DM}}}{m_a}\cos(m_a t + \varphi)$

Dark matter as a source for effective current $\Longrightarrow$ source magnetic field:

$$J_{\rm eff}(t) \sim g_{a\gamma\gamma}\, B_0(t)\,\sqrt{\rho_{\rm DM}}\,\cos m_a t \;\;\Longrightarrow\;\; B_a(t) \propto J_{\rm eff}(t)$$

Resonant Axion Searches

Axion-induce magnetic field induces an E.M.F.: $\mathcal{E}_a \sim V^{2/3}\,\partial_t B_a$

$$P_{\rm sig}^{(\rm r)} \sim \frac{\mathcal{E}_a^2}{R}\,\min\left(1, \frac{\tau_a}{\tau_{\rm r}}\right) \sim \omega_{\rm sig}^2\, B_a^2 V\,\min\left(\frac{Q_{\rm r}}{\omega_{\rm sig}}, \frac{Q_a}{m_a}\right)$$

$1/\tau_a \sim m_a \langle v^2 \rangle$

$1/\tau_{\rm r} \sim \omega_{\rm sig}/Q_{\rm r}$

$Q_a \sim 1/\langle v^2 \rangle$

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Maximise: $\omega_{\rm sig}$, B_a, V

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WARNING

QUANTITIES OFTEN LINKED

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Able to access small masses, but length-ratio suppressed

Different approach: Resonant Axion Searches

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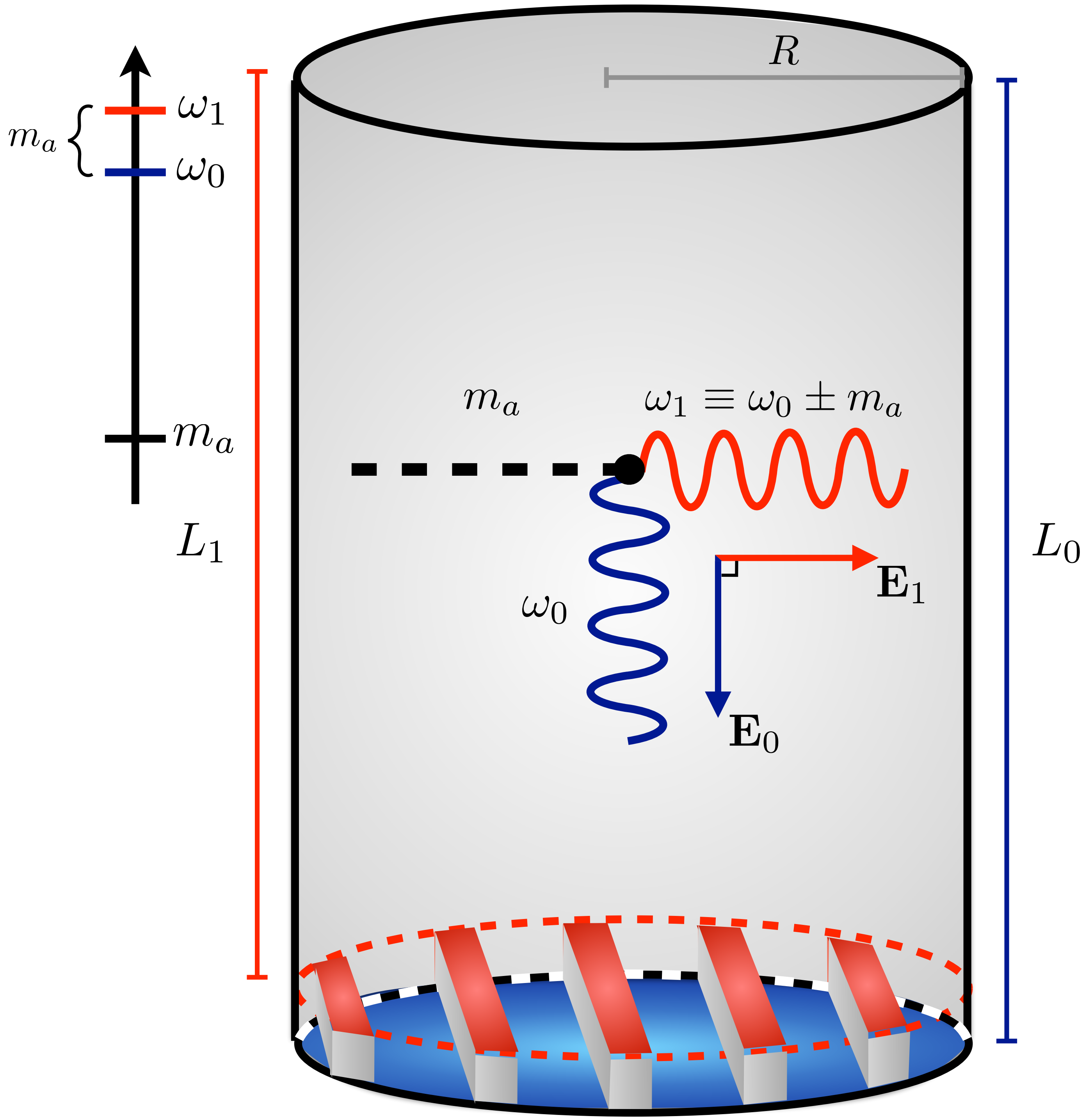
WARNING

Noise & exp. parameters not discussed yet

Comparison

	Static-field Haloscope	LC Resonator	RF Frequency Conversion
$J_{\rm eff}$	$\propto B_0^{\rm static}\cos(m_a t)$	$\propto B_0^{\rm static}\cos(m_a t)$	$\propto B_0^{\rm RF}\cos(\omega_0 \pm m_a)t$
$\mathcal{E}_a$	$\propto m_a/\omega_{\rm sig} \sim 1$	$\propto m_a V^{1/3} \lesssim 1$	$\propto (\omega_0 \pm m_a)/\omega_{\rm sig} \sim 1$
$P_{\rm sig}$	$J_{\rm eff}^2 V \min\left(\frac{Q_{\rm r}}{m_a}, \frac{Q_a}{m_a}\right)$	$J_{\rm eff}^2 m_a^2 V^{5/3} \min\left(\frac{Q_{\rm LC}}{m_a}, \frac{Q_a}{m_a}\right)$	$J_{\rm eff}^2 V \min\left(\frac{Q_{\rm SRF}}{\omega_0 \pm m_a}, \frac{Q_a}{m_a}\right)$

Axion Resonant Frequency Conversion



Superconducting RF Cavity

$$\omega_1 \sim 2\pi\,\mathrm{GHz}$$

$$Q_{\mathrm{int}} \sim 10^9 \div 10^{13}$$

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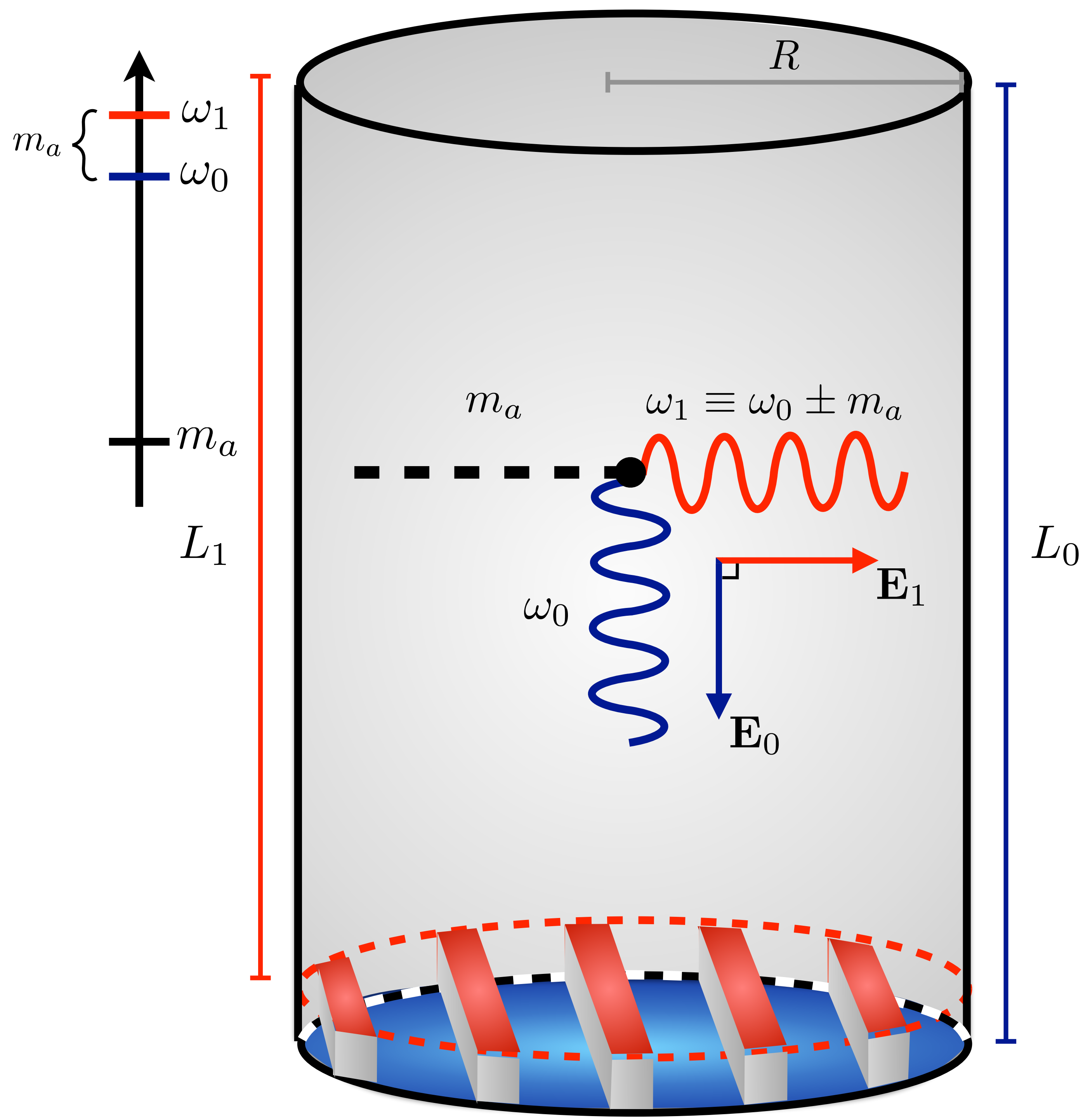


Superconducting RF Cavity

$$\omega_1 \sim 2\pi \,\mathrm{GHz}$$

$$Q_{\rm int} \sim 10^9 \div 10^{13}$$

Tunability:

$\delta\omega \lesssim \mathrm{MHz}$ piezos

$\delta\omega \gtrsim \mathrm{MHz}$ fins

Axion Signal

Signal Power Spectral Density (PSD):

$$S_{\rm sig}(\omega) = \frac{\omega_1}{Q_1}\,(g_{a\gamma\gamma}\,\eta_{10}\,B_0)^2\,V\,\frac{\omega^2}{(\omega^2-\omega_1^2)^2+(\omega\,\omega_1/Q_1)^2}\int\frac{d\omega'}{(2\pi)^2}\,(\omega'-\omega)^2\,S_{b_0}(\omega')\,S_a(\omega-\omega')$$

Axion PSD:

$$\langle a(t)^2\rangle = \frac{1}{(2\pi)^2}\int d\omega\;S_a(\omega) = \frac{\rho_{\rm DM}}{m_a^2}$$

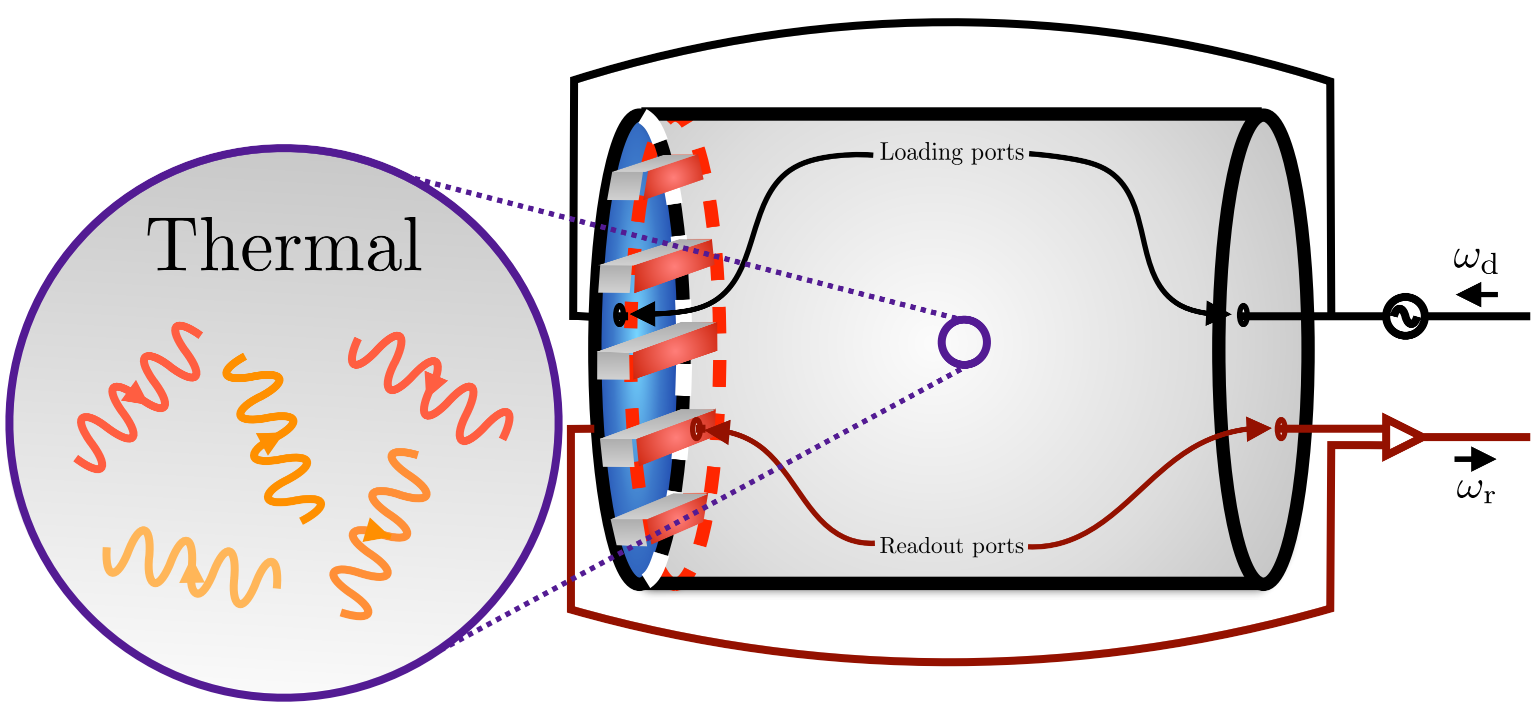
Power for monochromatic background field:

$$P_{\rm sig} \simeq \frac{1}{4}\,(g_{a\gamma\gamma}\,\eta_{10}\,B_0)^2\;\rho_{\rm DM}\,V\times\begin{cases} Q_1/\omega_1 & \frac{m_a}{Q_a}\ll\frac{\omega_1}{Q_1} \\ \pi Q_a/m_a & \frac{m_a}{Q_a}\gg\frac{\omega_1}{Q_1}\,, \end{cases}$$

Standard Noise Sources: Thermal Noise

Power Spectral Density:

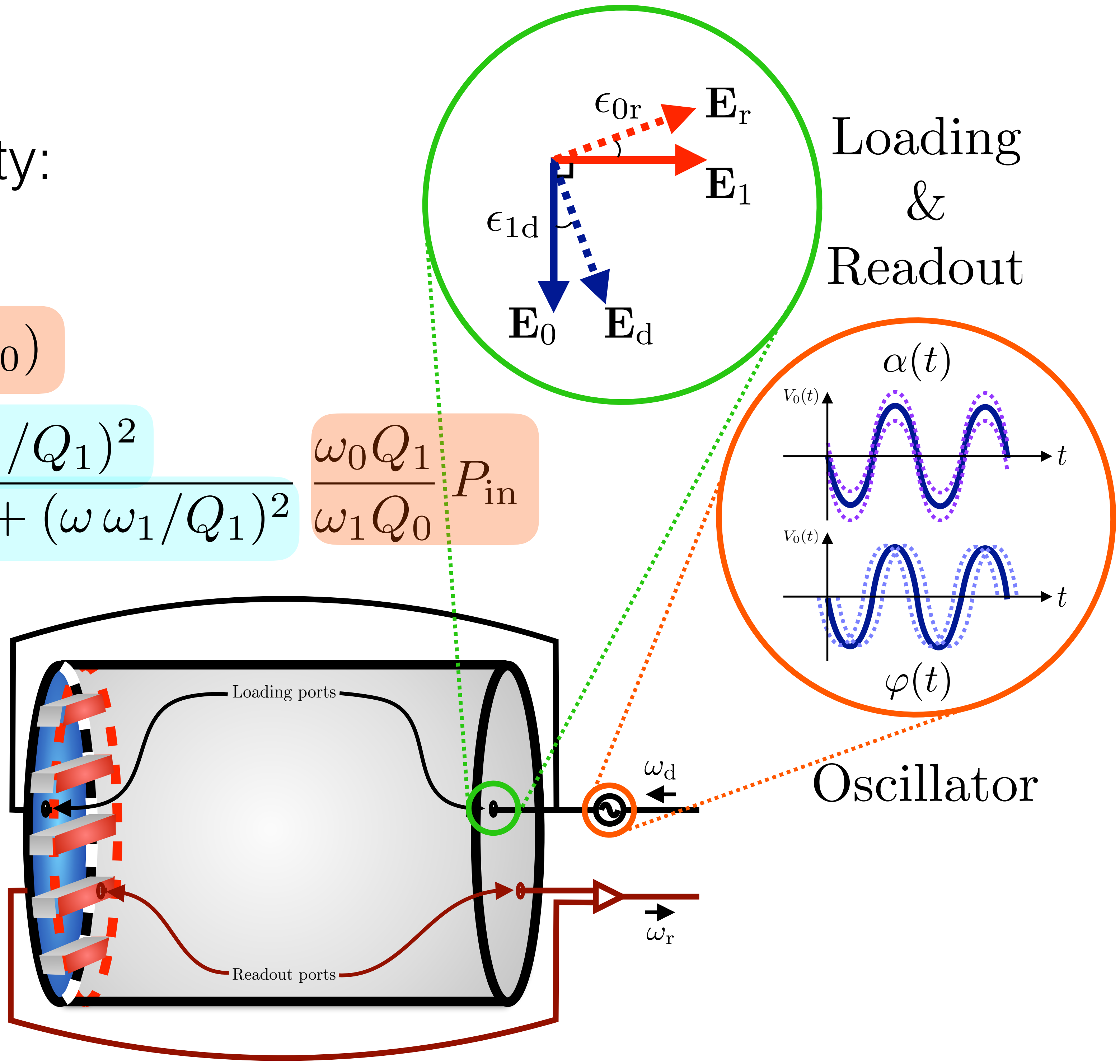
$$S_{\rm th}(\omega) = \frac{Q_1}{Q_{\rm int}} \frac{4\pi T\,(\omega\,\omega_1/Q_1)^2}{(\omega^2-\omega_1^2)^2+(\omega\,\omega_1/Q_1)^2}$$



Non-standard Noise Sources: Phase Noise

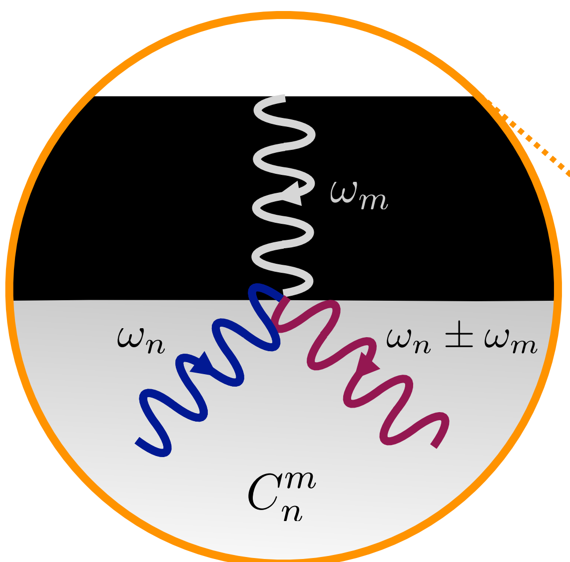
Power Spectral Density:

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2}\,\epsilon_{1\text{d}}^2\, S_\varphi(\omega-\omega_0) \times \frac{(\omega\,\omega_1/Q_1)^2}{(\omega^2-\omega_1^2)^2+(\omega\,\omega_1/Q_1)^2}\,\frac{\omega_0 Q_1}{\omega_1 Q_0}\,P_{\text{in}}$$



Non-standard Noise Sources: Wall Vibrations

Vibrations



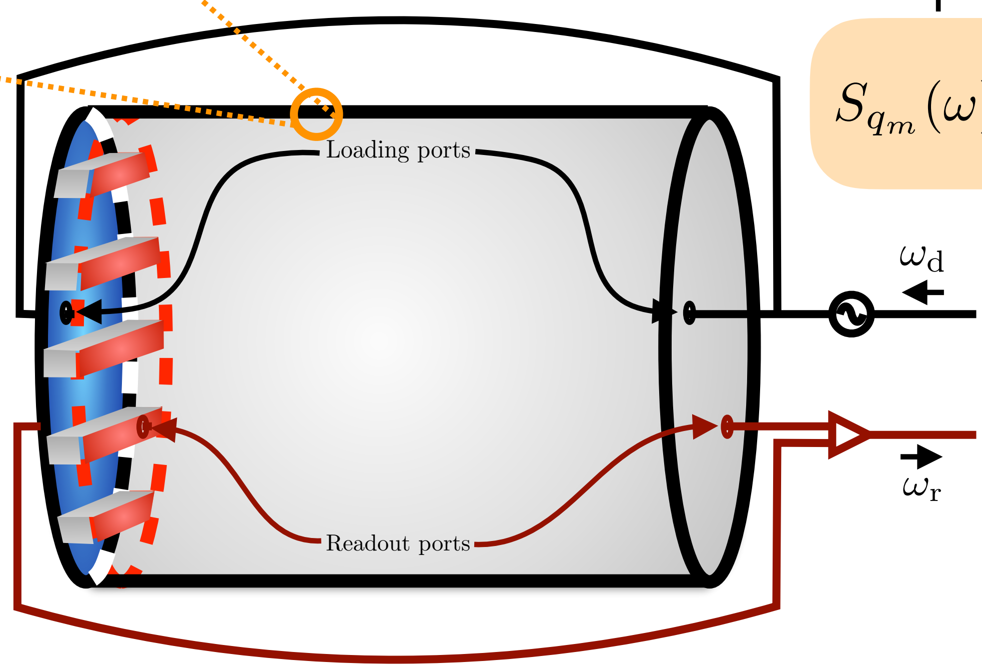
Power Spectral Density:

$$S_{\text{mech}}(\omega) \simeq \frac{\epsilon_{1d}^2}{4}\,\frac{\omega_0}{Q_0}\,P_{\text{in}} \times$$

$$\sum_{n=0,1} \frac{\left(S_{q_m}(\omega-\omega_0)/V^{2/3}\right)(\omega_n/Q_n)\,\omega_n^4\,\omega^2}{\left[(\omega^2-\omega_n^2)^2+(\omega\,\omega_n/Q_n)^2\right]\left[(\omega_0^2-\omega_n^2)^2+(\omega_0\,\omega_n/Q_n)^2\right]}$$

Displacement PSD:

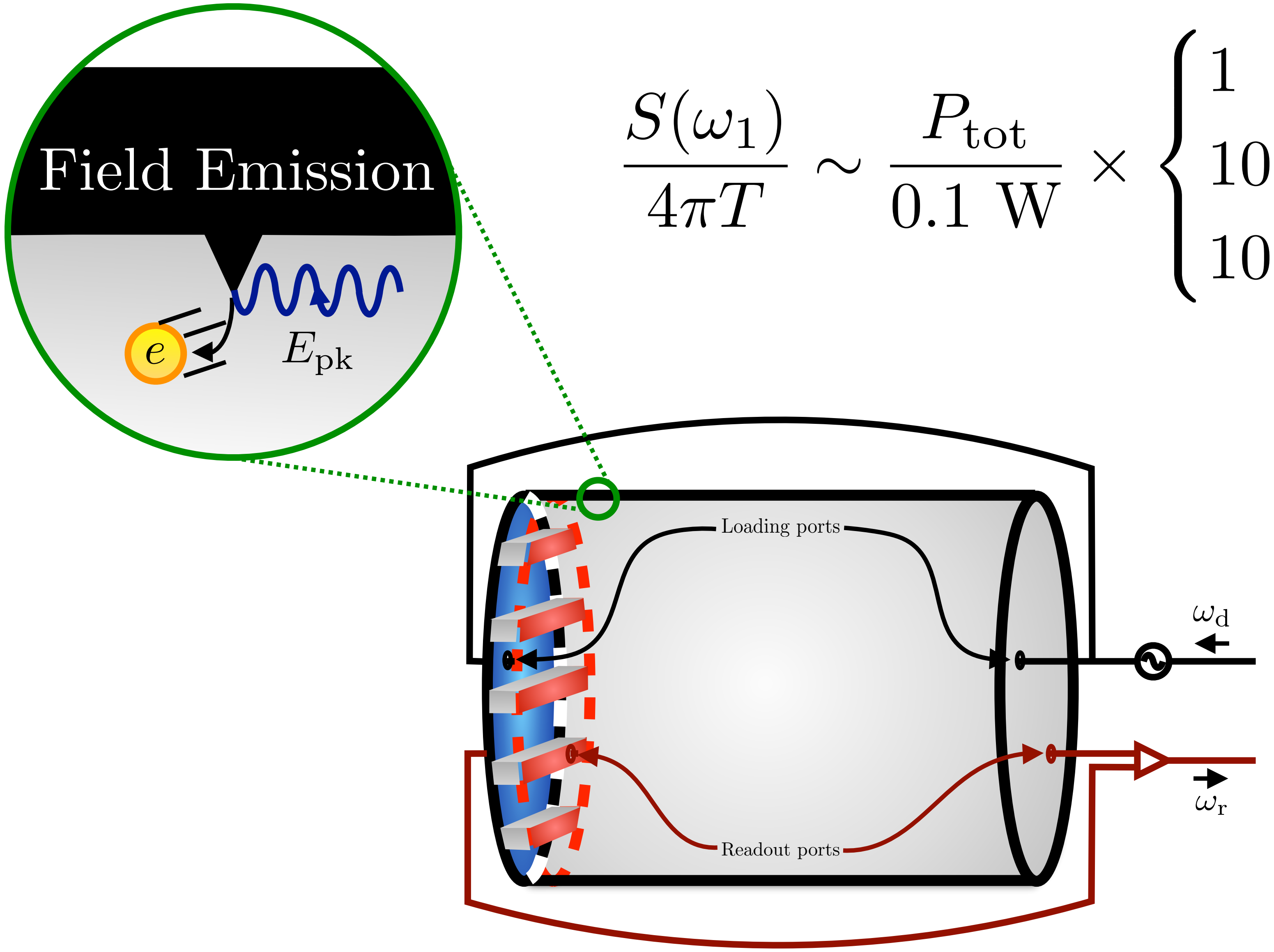
$$S_{q_m}(\omega) \simeq \frac{1}{M^2}\,\frac{S_{f_m}(\omega)}{(\omega^2-\omega_m^2)^2+(\omega_m\omega/Q_m)^2}$$



Non-standard Noise Sources: Field Emission

Power Spectral Density:

$$\frac{S(\omega_1)}{4\pi T} \sim \frac{P_{\rm tot}}{0.1\ {\rm W}} \times \begin{cases} 1 & \text{synchrotron} \\ 10^{-6} & \text{transition} \\ 10^{-5} & \text{Bremsstrahlung}\,, \end{cases}$$



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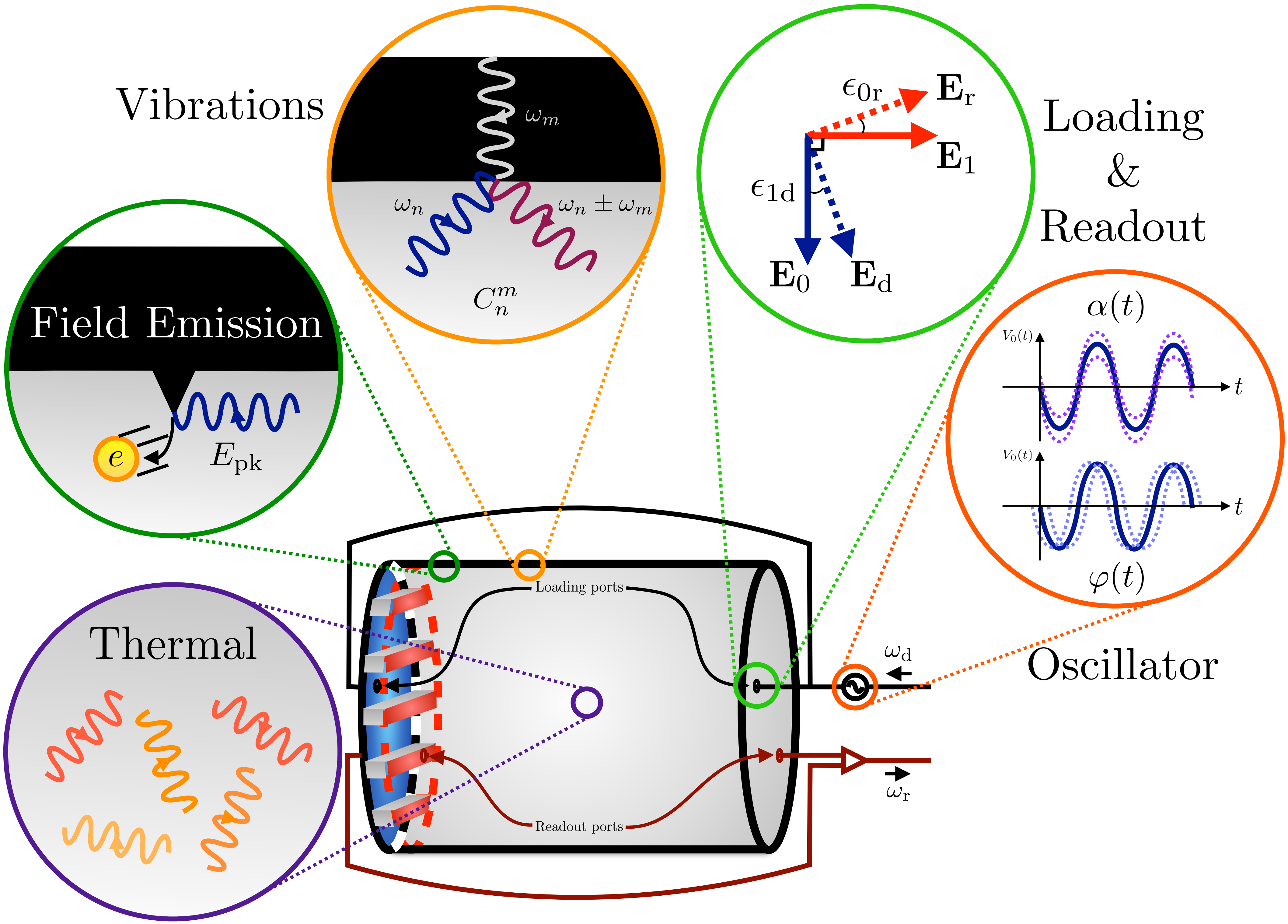

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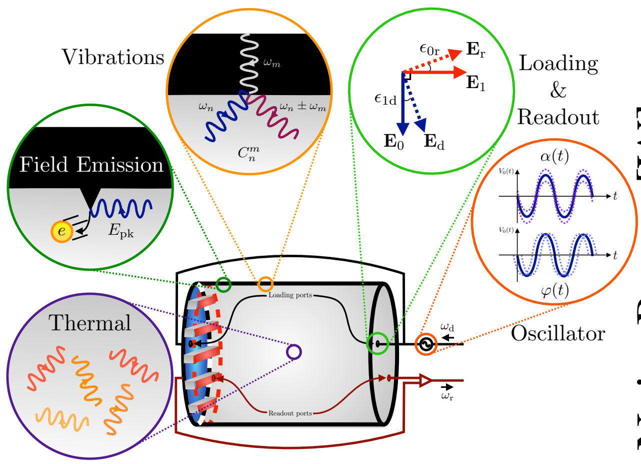
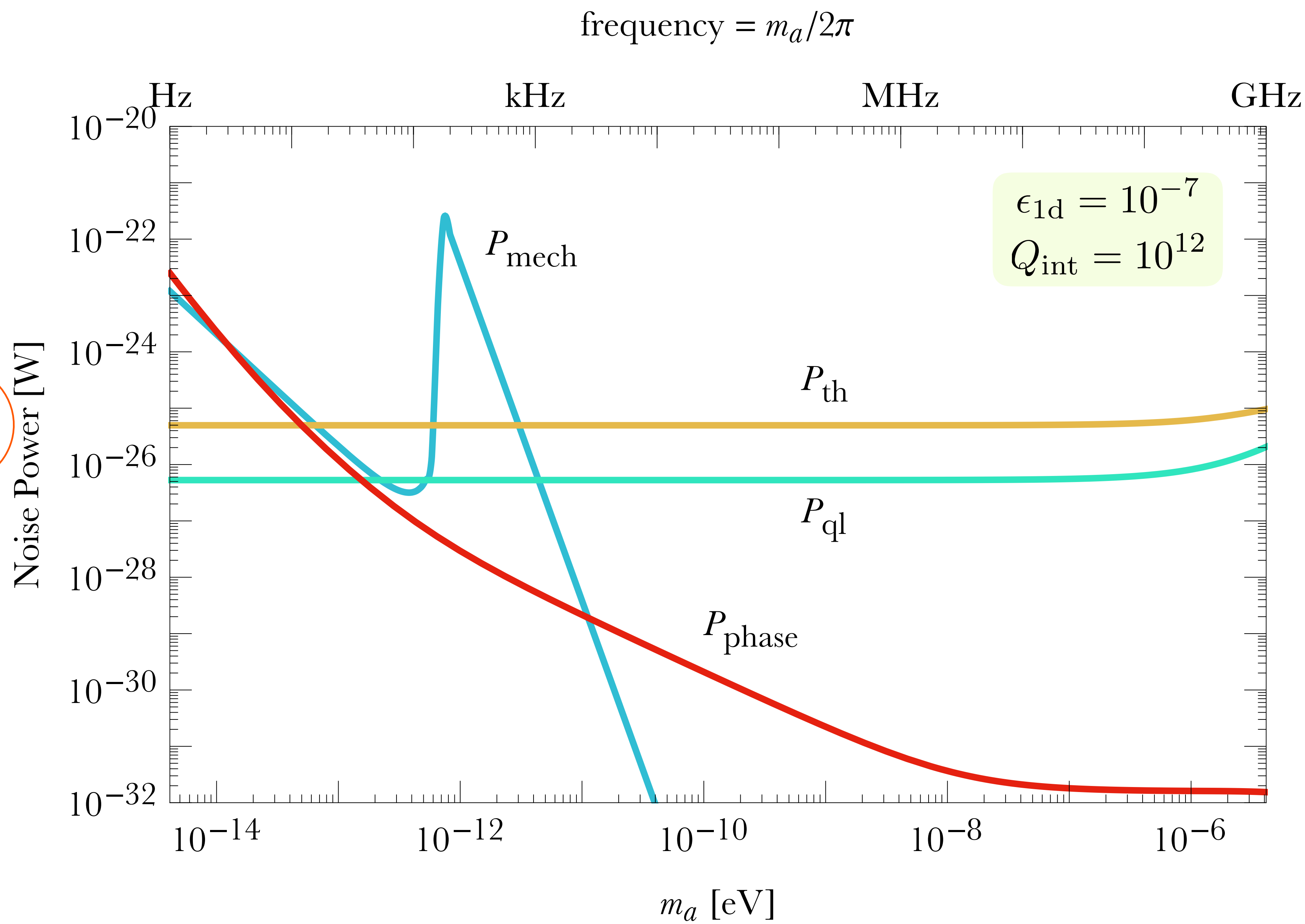
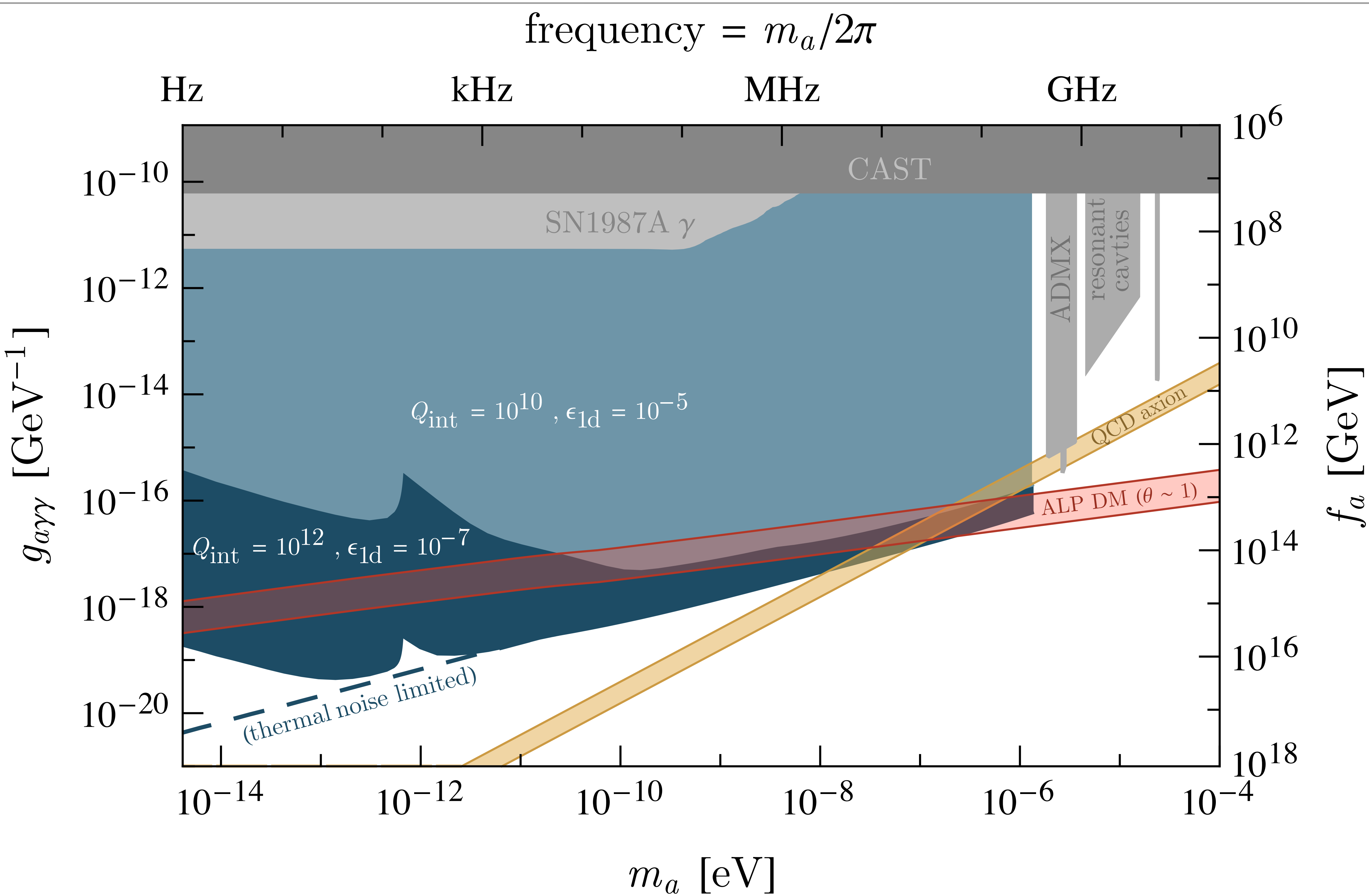




Potential Sensitivity



Outlook

Radio-Frequency **up-conversion** approach

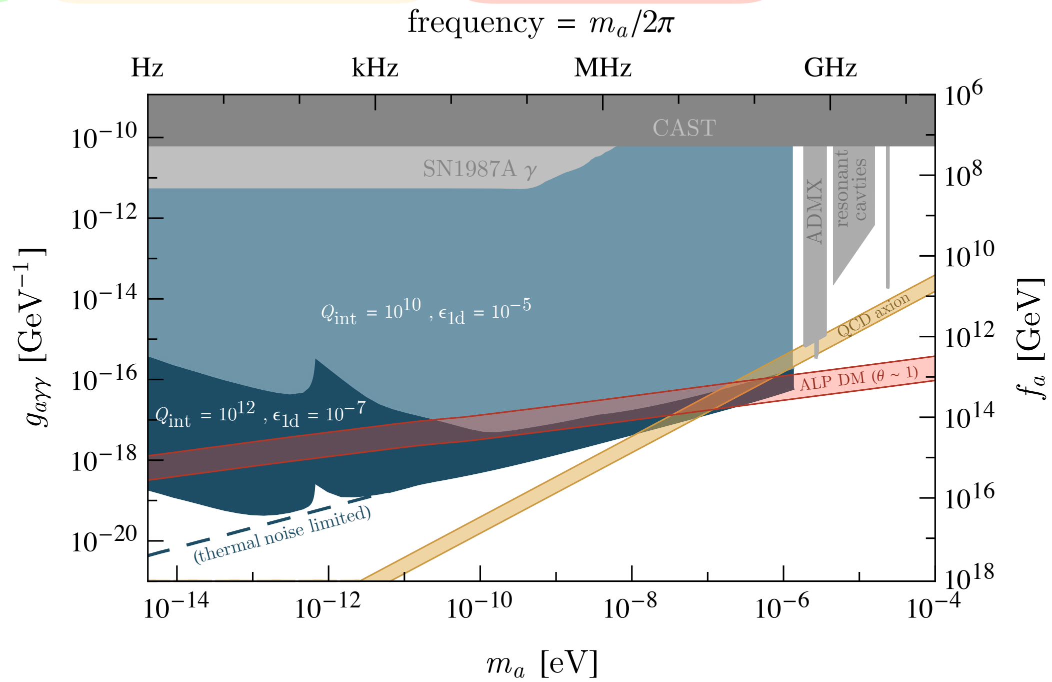
$$\omega_{\rm sig} = \omega_0 \pm m_a$$

Parametric gain for small axion masses vs. LC Resonator

$$\frac{\rm SNR}{\rm SNR^{LC}} \sim \frac{\omega_0 \pm m_a}{m_a}\left(\frac{Q_{\rm int}}{Q_{\rm LC}}\right)^{1/2}\left(\frac{T_{\rm LC}}{T}\right)^{1/2}\left(\frac{B_0}{B_{\rm LC}}\right)^{2}$$

SLAC group beginning prelim. cavity studies

CERN & FNAL SRF groups voiced interest



Backup

A ⚡ introduction to Axions

QCD has a CP problem:

$$\mathcal{L} \supset \frac{\bar{\theta} g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a}$$

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Term violates CP — leads to neutron EDM

$$d_n \sim 10^{-16} \bar{\theta} \; e \; \mathrm{cm}$$

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Experimental limit:

$$d_n^{\exp} \lesssim 10^{-26} \; e \text{ cm}$$

$$\boxed{\bar{\theta} \lesssim 10^{-10}}$$

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Solution: $U(1)_{PQ}$ symmetry anomalous under QCD: pNGB after instanton breaking — QCD axion!

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Weinberg (1978)
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Potential for axion generated by confinement:

$$V = -m_\pi^2 f_\pi^2 \left(1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2 f_a} + \frac{\bar{\theta}}{2}\right)\right)^{1/2}$$

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Axion mass related to QCD scale: $m_a^2 f_a^2 \sim m_\pi^2 f_\pi^2$

A ⚡ introduction to Axions

Axion-like particles (ALPs)

$$\mathcal{L}_{\rm ALP} \supset \frac{1}{2} m_a^2 a^2 + \mathcal{L}_{\rm int}$$

Generic shift-symmetric P-odd scalar field w/ derivative couplings to SM fields

Motivations:

a) One of ~few concrete predictions from known String compactifications (string axiverse)
b) ALPs as Dark Matter from misalignment
c) Technology to search for ALPs exists

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Arvanitaki et al (2009)
Stott et al (2017)
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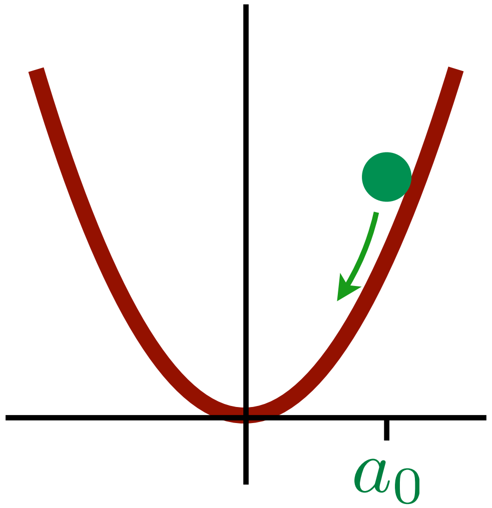
ALPs as Dark Matter: Misalignment

Axion EoM in FRW Universe: $\ddot{a} + 3H\dot{a} + m_a^2 a = 0$

$$3H > m_a, \quad a = a_0$$

$$3H \lesssim m_a, \quad a \simeq a_0 \left(\frac{\alpha(H = 3m_a)}{\alpha(t)} \right)^{3/2} \cos(m_a t + \varphi)$$



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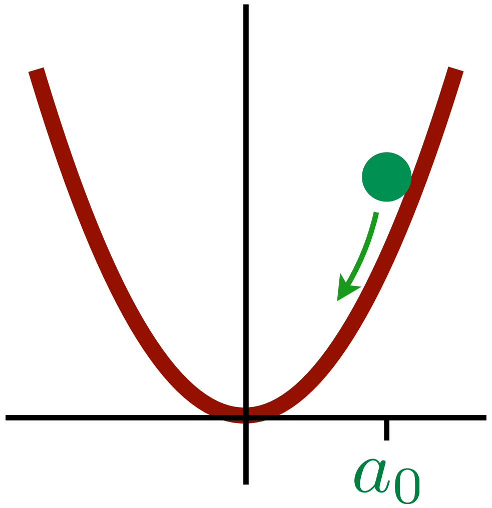

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DM energy density: $\rho_{\rm DM} \sim T^3 T_{\rm eq}$

Recall: $\rho_a \sim m_a^2 a_0^2 \qquad T \sim (H^2 m_{\rm P}^2)^{1/4}$

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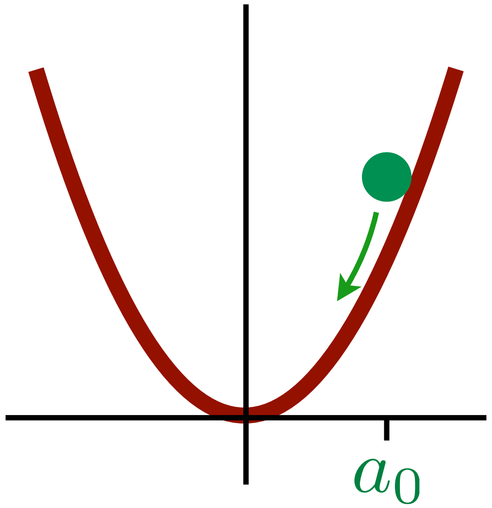


DM energy density: $\rho_{\mathrm{DM}} \sim T^3 T_{\mathrm{eq}}$

Recall: $\rho_a \sim m_a^2 a_0^2 \qquad T \sim (H^2 m_{\mathrm{P}}^2)^{1/4}$

Relic abundance: $a_0^2 = \left(\frac{T_{\mathrm{eq}}^2 m_{\mathrm{P}}^3}{m_a}\right)^{1/2}$ $\qquad a_0 = \theta_0 f_a$

Power comparison with static LC resonator

Power for monochromatic background field:

$$P_{\rm sig} \simeq \frac{1}{4}\left(g_{a\gamma\gamma}\,\eta_{10}\,B_0\right)^2\,\rho_{\rm DM}\,V \times \begin{cases} Q_1/\omega_1 & \frac{m_a}{Q_a} \ll \frac{\omega_1}{Q_1} \\ \pi Q_a/m_a & \frac{m_a}{Q_a} \gg \frac{\omega_1}{Q_1} \end{cases},$$

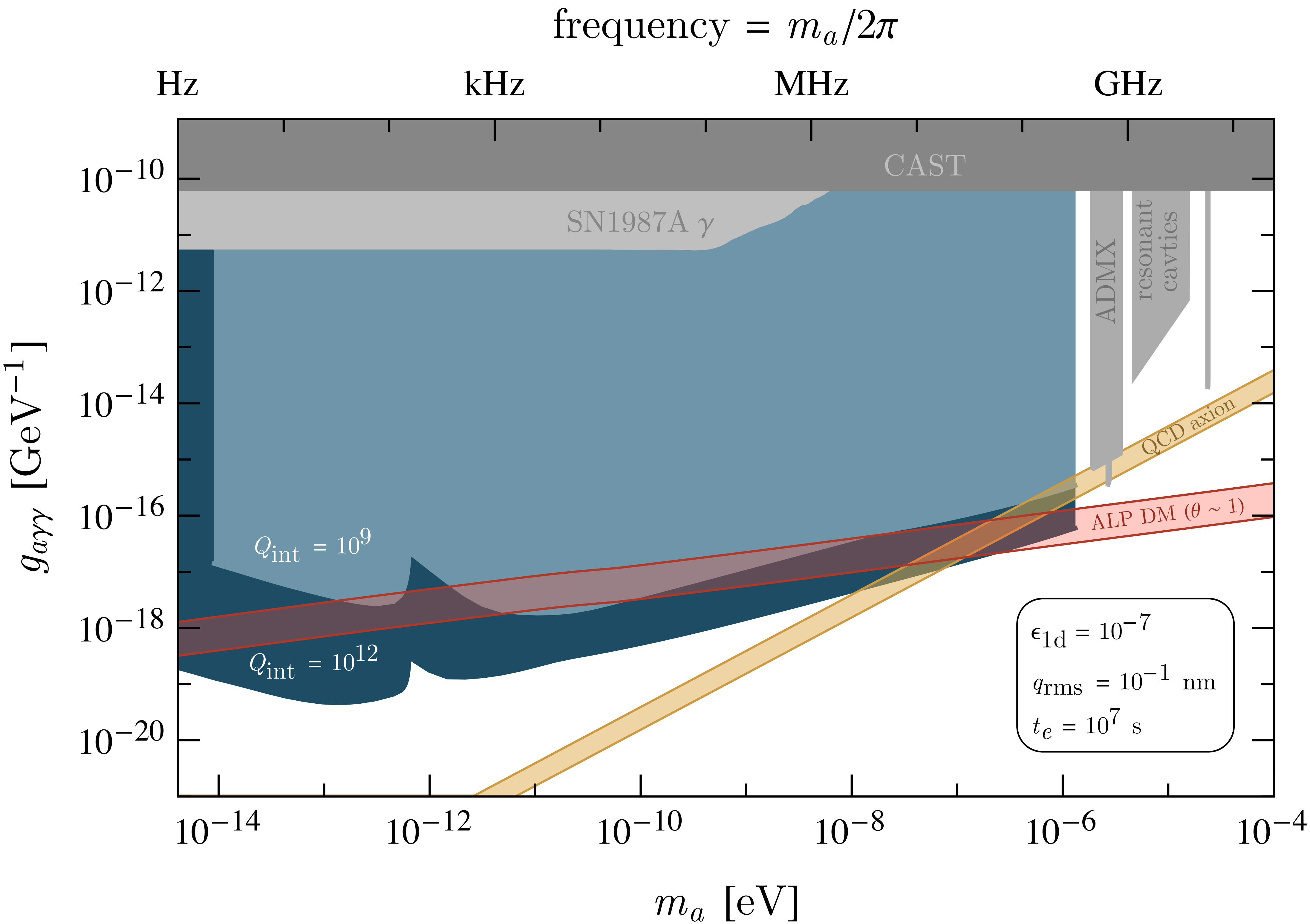
Power for LC resonator:

$$P_{\rm sig}^{\rm (LC)} \sim \left(g_{a\gamma\gamma} B_{\rm LC}\right)^2 \rho_{\rm DM}\, V^{5/3} \min(Q_{\rm LC}, Q_a)\, m_a$$

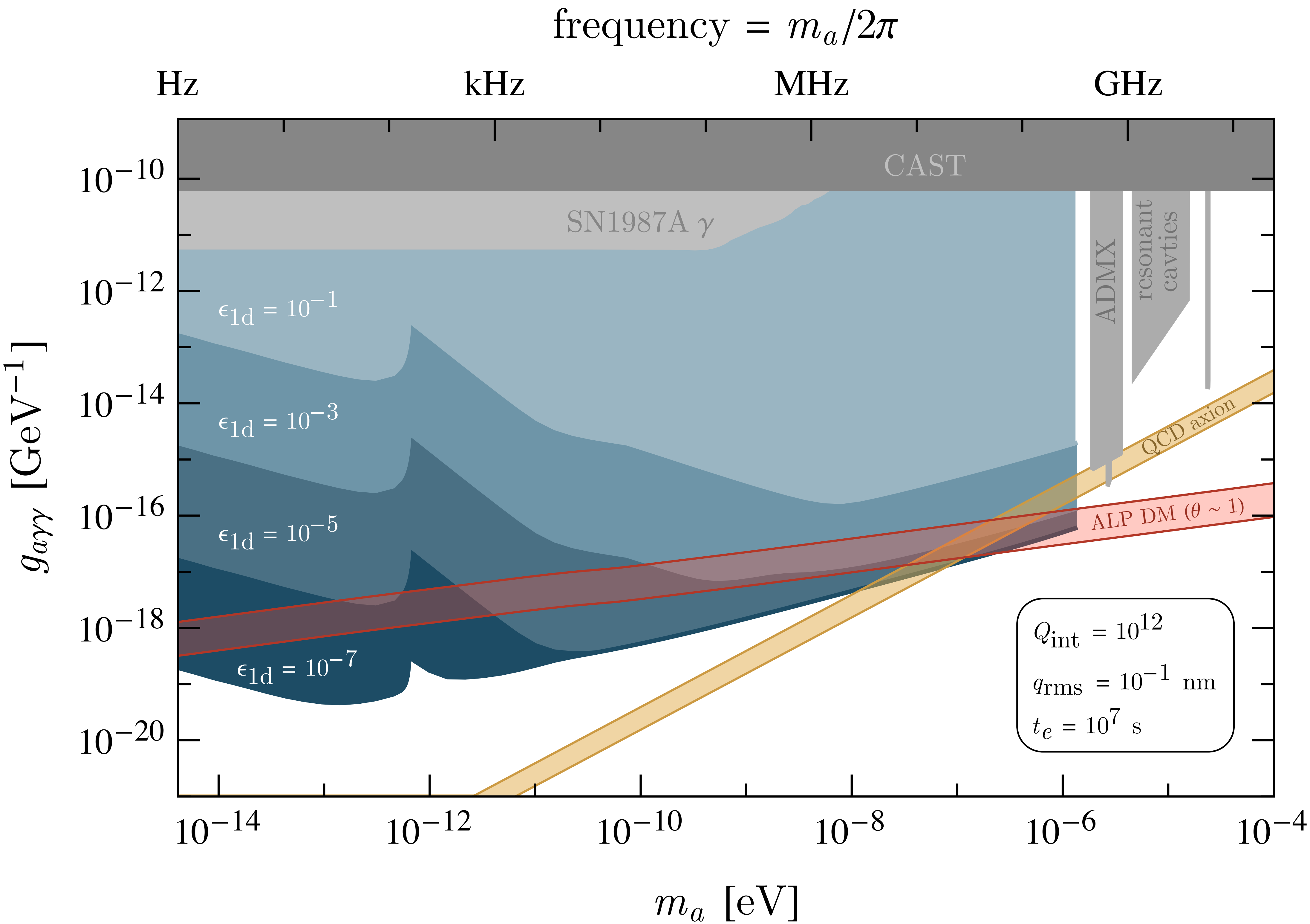
Ratio:

$$\frac{P_{\rm sig}}{P_{\rm sig}^{\rm (LC)}} \sim \left(\frac{0.2\ \rm T}{4\ \rm T}\right)^2 \times \begin{cases} \left(Q_1/Q_a\right)^2 \frac{(\omega_1/Q_1)}{(m_a/Q_a)} & \frac{m_a}{Q_a} \ll \frac{\omega_1}{Q_1} \\ \left(\omega_1/m_a\right)^2 & \frac{m_a}{Q_a} \gg \frac{\omega_1}{Q_1} \end{cases}$$

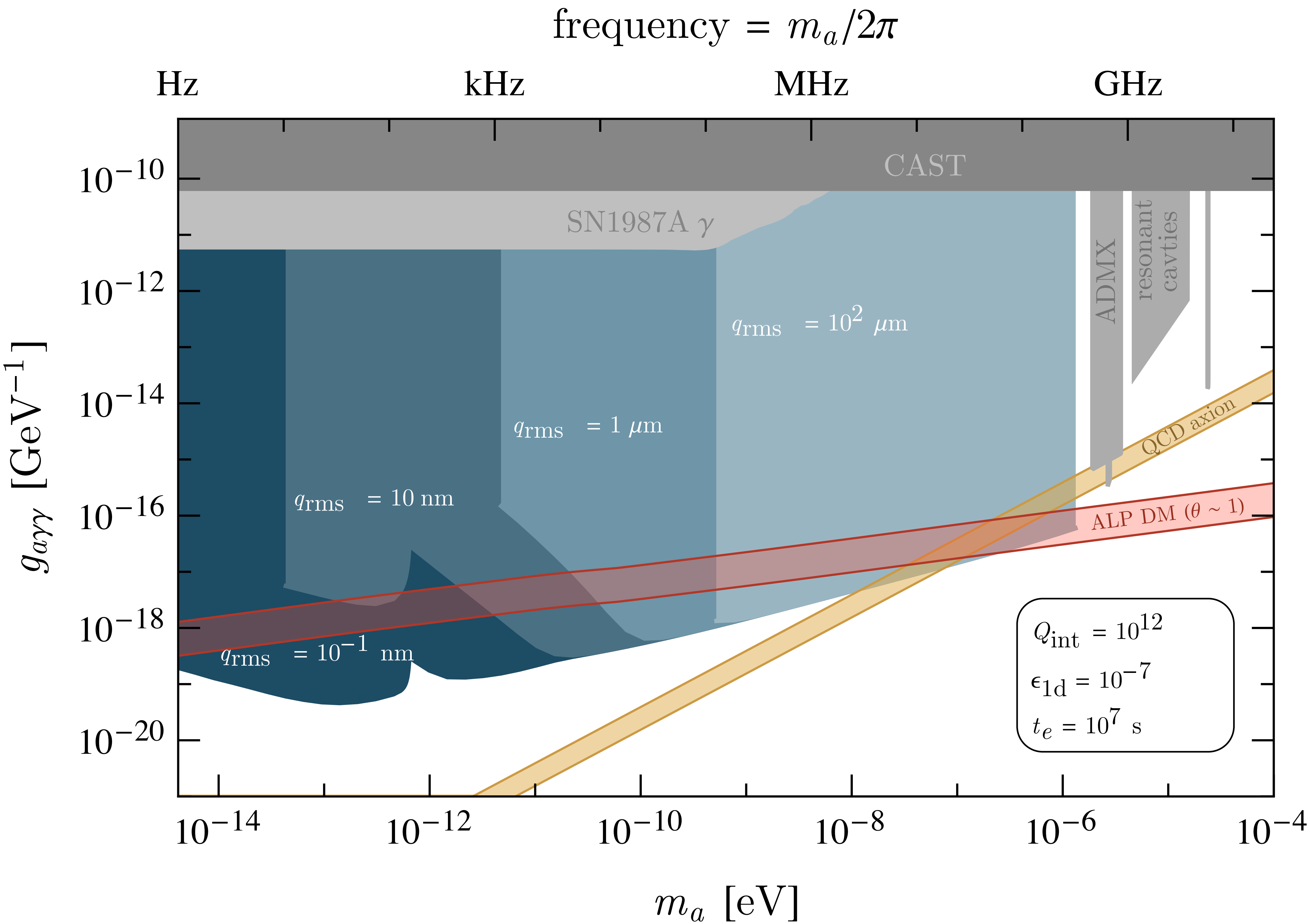
Potential Sensitivity dependences — $Q_{\rm int}$



Potential Sensitivity dependences — geom. factor



Potential Sensitivity dependences — wall disp.



Potential Sensitivity dependences — wall disp.